

Mathematicks made Plain, In the Solution of Variety Of Useful Propositions

I N

*Arithmetick,
Interest, Simple and
Compound.
Geometry.
Surveying.
Gauging.
Measuring all Artificers
Works.
Trigonometry, Plain
and Spherical.
Astronomy.
Geography.*

*Navigation.
Dialling.
Chronology.
Dioptricks.
Statics.
Gunnery.
Military Fortification.
Civil and Naval Archi-
tecture.
Mechanick Powers.
Automaticks, or Clock
Work, &c.*

All Perform'd by that Excellent Line of Num-
bers, commonly call'd *Gunter's-Line*.

B E I N G

A necessary Companion for Gentlemen, Mi-
litary Officers, Engineers, Architects, Seamen, Sur-
veyors, Usurers, Shopkeepers of all sorts; Artificers,
as Masons, Bricklayers, Carpenters, Shipwrights,
Watch-makers, Glaziers, Painters, Plasterers, &c.

By *Richard Neve*, Philomath.

Approv'd of by the Royal Society.

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and *T. Ballard*, all in Little-Britain. 1702. Pr. 2 s.

To the Worshipful

JOHN FULLER

O F

Brightling in the County of Sussex, Esq; one of Her Majesties Justices of the Peace for the said County.

Worthy Sir,

I *Am bound in Gratitude, to present you with this Abridgment of Practicall Mathematicks; and that for these particular Reasons. First, Because, upon sight of the Copy of this Book, your Worship was pleased to shew it to the Royal Society, and from them to bring me their Approbation of it. Secondly, Because, you are a general encourager of all Ingenious Stu-*

The Dedication.

dies. And, Thirdly, Because you have exprest a particular Respect to me by your many Civilities; notwithstanding I was an absolute Stranger to your Worship, till within these few Months. And therefore I think my self bound to an humble Acknowledgment of your Favours; which, Sir, if you will accept this as a Token of, it will yet more Oblige him, who is already,

Your Worship's

Most Humble, and

Most Obliged Servant,

Richard Neve.



TO THE
READER.

THE following Sheets have been
seen by several of my Friends,
who judged them fit for the
Publick; what the rest of the World may
think of them, I know not, for I
know there are a sort of Men in the
World, that are readier to find Faults
than to mend them; to such, I shall
only offer that of the Poet,

Carpere vel noli Nostra, vel ede tua.

Or as it may be Englished,

Either commend me, or come and mend me.

The Preface.

I am not ignorant, that many good Authors have already treated of this Subject; as Mr. Gunter, Mr. Wingate, Mr. Collins, Mr. Oughtred, Mr. Partridge, Mr. Leybourn, Mr. Brown, and others, too many to mention here; and therefore what I have here done may seem altogether Superfluous: But let me add, That many of these Authors have treated only occasionally of this amongst other Subjects; whereby their Volumns are rendred too large for the Pocket, and therefore (unless a Man can carry all their Rules in his Head) they lose the *End* for which *this* was compos'd.

Again, some of these Authors have treated of the Line of Numbers as apply'd to Circles, or Quadrants, or to slide one by another: In these Cases a Man must have those particular Instruments, otherwise nothing is to be done.

'Tis true, some of these Authors have treated of it (as I have done) as 'tis commonly made on the Carpenters Rule, to be us'd with Compasses: And what they have done, they have done well enough;

The Preface.

enough ; only they have enlarged too much upon *some* Particulars, whereby they were forced to omit *others* as useful ; and so could not sufficiently display the Excellency of the Line.

Having thus far shewn what has been done by *others*, give me leave to tell thee what I have (here) done my self.

First then, I have (in the following Sheets) shewn how (with a common Carpenters Rule, and a pair of Compasses) to solve all the most usual and most useful Propositions, in all these Arts and Sciences mention'd above in the Title Page.

The Number of Propositions, contain'd in this Book, are almost incredible, amounting to near *Seventeen Thousand*, which is perhaps almost 100 times as many, as ever was mentioned by any of (or all) the other Authors that have treated of this Subject : And yet I dare presume boldly to affirm, That there is not one of them all, but what is (*here*) made as intelligible, (even to the meanest Capacity, that has but learnt the Terms of Art,) as by any other Author.

The Preface.

I have taken a great deal of Pains, to comprize so much in so little Compass; for thou hast here an Epitome of the whole Practical Mathematicks; and in order to this, I have Calculated several Hundreds of fixed Numbers, which may serve as Centers on the Line, for the more easy and expeditiously Solving of many Problems.

A particular Account of all that is contain'd in this Treatise may be seen in the Proeme to each Book.

The whole is compris'd in so brief, and yet so plain a Method, that I think I may boldly affirm, *The like (of this Nature) was never before extant.* And let me add, That he that has this Book in his Pocket, together with a common Carpenters Rule and a pair of Compasses) may solve almost all Propositions in the Mathematicks, with great ease and expedition, and sufficient Accuracy. I shall only add,

Reader, the Printer's Faults are none of mine, Correct the Errors, else the Fault is thine.

Et sic Vale.

T H E

T H E
C O N T E N T S.

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MATHE-

MATHEMATICS

Made plain,
In the Solution of Variety of Useful
Propositions in *Arithmetick*, &c.

BOOK I

*Of the Use of the Line of Numbers
in Arithmetick.*

PROEME

I Shall not here insist upon the making of the Line of Numbers; that being a thing so well known to all Mathematical Instrument Makers; (and any one else may buy them cheaper than he can make them himself;) but shall hasten to shew the Excellency and Usefulness of it.

CHAP. I.

Of Numeration on the Line of Numbers.

Numeration is the Art of reading Numbers; i. e. of relating the true Value of any Number, as it is express'd, either by proper Characters or Figures, as in Figurative Arithmetick; or by Divisions and Subdivisions, as in our present Case of Instrumental Arithmetick.

Therefore, in order to teach the Art of Numeration on the Line, it will be necessary to shew how, or in what manner the Line is divided and numbred; as also what those Divisions and Numbers set to them upon the Rule, do signify.

Know therefore, that at the Beginning of the Line, there is set the Figure 1; and so proceeding successively from 1 to 2, 3, 4, 5, 6, 7, 8, 9, to 1 again in the Middle of the Line; from whence it proceeds in the same Order to the other End, where it ends with 10.

The Reason why the Line is numbered only with Digits, is, that by prefixing or affixing of Cyphers to those Digits, it might be capable to express different Numbers, as there may be occasion, according to the Nature of the Question.

Thus, sometimes the 1, at the Beginning of the Line, signifies 1 only, and then the 1 in the Middle signifies 10, and the 1 at the other End 100.

Again, If 1 at the Beginning denotes 10, then 1 in the Middle is 100, and 1 at the End, 1000. So that for any Number between 10 and 1000, the Figures very readily direct you.

Again, If 1 at the Beginning be $\frac{1}{10}$ of an Integer, then 1 in the Middle is 1 Integer, and 1 at the End is 10.

Again, If 1 at the Beginning be $\frac{1}{100}$ of an Integer, then 1 in the Middle is $\frac{1}{10}$ of an Integer, and 1 at the End is 1 Integer.

Note also, That, as the whole Line is divided into 100 Parts, so is each of those Parts subdivided into 10 Parts more, or as the Length thereof will permit, more or less, in a Decuple or Ten-fold Order: and every 5th of the Grand Intermediate Divisions is drawn longer than the rest, for Ease in counting. This being premis'd, I proceed to shew in

PROP. I.

How to find the Point on the Line representing any given Number.

The RULE. How many Places soever your Number is of, for the first Figure of your Number, take the same Figure on

on the Line: for the second Figure of your Number, take the Number thereof on the grand (or longer) Intermediate Divisions on the Line: for the third Figure of your Number, take the Number thereof on the smaller Intermediate Divisions; and the place of the fourth (fifth, &c. if you have more) must be found by Estimation.

Note, By the first, second, third, &c. Figure, is meant the first, second, third, &c. Figure, from the Left Hand toward the Right. This being known, the Precept is plain, and needs no Example: yet, because the Learner ought to be very perfect in this Proposition, I shall here give him one or two Examples.

Example 1. Let it be requir'd to find the Place of 144 on the Line. For your first Figure 1, count the 1 in the Middle of the Line; for your second Figure 4, count 4 of the Grand Divisions, betwixt the Figures 1 and 2, on the Line; and for your third Figure 4, count 4 of the smallest Intermediate Divisions; and that very Point is the Place upon the Line representing 144.

Example 2. Let it be requir'd to find the Place of 1728 on the Line. For your first Figure 1, take 1 in the Middle of the Line; for your second Figure 7, take 7 of the Grand Divisions; for your third Figure 2, take 2 of the smaller Intermediate Divisions; and for your fourth Figure 8, take (by Estimation) 8 tenth Parts [that is a little better than three quarters] of one of the next of those smaller Intermediate Divisions; and that is the Place of 1728.

Now, by these two Examples, you may perceive, that the Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, do vary their Signification, according to the Nature of the Question. And by this Variation and Change of the Powers of these Numbers, from 1 to 10, 100, or 1000, &c. any Proportion, either Arithmetical or Geometrical, may

be wrought; one whereof I will here insert, (for your better Exercise of Numbring on the Line; by the often practice whereof, you will find the Work to be facile and delightful;) which shall be this following.

P. R. O P. II.

Having two Numbers given, to find as many more as you please, in continuall Proportion one to another, as the two given Numbers were.

T H E R U L E.

Place one Foot of the Compasses in the first given Number on the Line, and extend the other to the second given Number; then may you turn the Compasses (the same way) from that second Number to a third, from that third to a fourth, &c. to as many as you please. But when the Compasses run off the Line, place them on again at the same Number (they were at when they ran off) at the other End of the Line.

Example 1.

Let the two given Numbers be 1 and 2; Place one Foot of your Compasses in 1, and extend the other Foot to 2; then the Compasses being turn'd about, will reach from 2, to 4; and from 4, to 8; and from 8, to 16; and from 16, to 32, &c.

Example 2.

If the two given Numbers were 10 and 9, decreasing; then the Extent of the Compasses from 10 to 9, will reach from 9, to 8.1; and from thence to 7.29, and from thence to 6.56, &c.

Because the Line of Numbers is sometimes very erroneously made, it will not perhaps be altogether indifferent, if (before I conclude this Chapter) I insert this following Proposition.

P. R. O P.

P R O P. III.

To find whether the Line of Numbers be truly made, or not.

To do this, take a very sharp-pointed Pair of Compasses, and extend their Points (as nicely as you can) from 1, to 2; on the Line; then turning the Compasses about, they shall (if the Line be truly made) reach from 2, to 4; and from 4, to 8; and from 8, to 16; and from 16, to 32; and from 32, to 64, &c.

Also, the same Extent of the Compasses shall reach exactly (if the Line be truly made) from 1.5, to 3, and from thence to 6, and from thence to 12, and from thence to 24, and from thence to 48, and from thence to 96, &c.

Again, the same Extent of the Compasses shall reach exactly (if the Line be truly made) from 2.5, to 5; from 5, to 10; from 10, to 20; from 20, to 40; and from 40, to 80, &c.

Likewise, the same Extent of the Compasses shall reach exactly (if the Line be truly made) from 3.5, to 7; from 7, to 14; from 14, to 28; and from 28, to 56, &c.

In like manner, the same Extent of the Compasses shall reach exactly (if the Line be truly made) from 4.5, to 9; from 9, to 18; from 18, to 36; and from 36, to 72, &c.

Or, you may extend the Compasses from 1, to 3; and then (if the Line be truly made) that Extent shall reach (exactly) from 3, to 9; from 9, to 27; and from 27, to 81, &c.

By these and such like Ways, you may easily try whether or no your Line of Numbers be exactly made.

G I H A B . 3 H.

Multiplication by the Line.

Multiplication is the next Rule, in the Order of our Instrumental Arithmetick; because Addition and Subtraction can't be wrought by the Line of Numbers.

Multiplication is a Rule, by which we find the Increase or Amount of any Number, being so many times taken as there are Units in another Number: and this Increase or Amount is call'd the Product; and the two Numbers producing it are call'd the Factors.

The Reader is here to Note, That almost all Operations on the Line of Numbers, are but the working so many Analogies or Proportions: And therefore if any thing seem difficult to the Learner, (either in this or the three following Chapters) let him consult Chap. VI. of this Book. This being premis'd, I come to shew how to perform Multiplication by the Line. To do which, this is

The ANALOGY.

As 1, to any one of the Factors; so is the other Factor, to the Product.

Therefore,

Extend the Compasses from 1, to any one of the Factors; (or Numbers to be Multiply'd;) and the same Extent of the Compasses, will reach (being apply'd the same way) from the other Factor, to the Product.

Example.

Let it be requir'd to multiply 19 by 5. Here, the Extent from 1 to 5, will reach from 19 to 95, the Product sought.

This is too plain to need another Example. And here let me inform the Reader, That I shall be cautious (throughout this whole Book) how I multiply needless

Exam-

Chap. II.

Division

Examples, not only (tho chiefly) for Brevity sake, but because any one may frame Examples at his pleasure, which he may prove with his Pen. Therefore I shall only add two or three Notes, and so conclude this Chapter.

Note 1. It matters not where you take your *One*, whether at the Beginning or Middle of the Line; Or, if your Compasses be too short to reach forward from 1, at the Beginning or Middle of the Line, you may extend them backward from 1, at the Middle or End: provided that you know what Denomination to give to your Product; which you may do by

Note 2. The Product always contains so many Figures as are contain'd in both the Factors; unless the two first Figures (from the Left towards the Right) of the Product be greater than the two first Figures of any of the Factors; for if so, the Product must have one Figure less than is contain'd in both the Factors.

Note 3. If the Compasses run off the Line, Note the Point they stood at when they ran off; and set them on again at a like Point in the other Radius of the Line. This Note ought to be well understood, it being of general Use in all Propositions, wherein the Compasses may happen to run off the Line.

CHAP. III.

Division by the Line.

Division is a Rule by which we discover how many times one Number (call'd the Divisor) may be had in another (call'd the Dividend,) and the Answer found is call'd the Quotient. To work this by the Line, this is

The ANALOGY.

As the Divisor, is to the Dividend; so is 1, to the Quotient.

Or,

Or,

As the Divisor, is to 1; so is the Dividend, to the Quotient.

Therefore,

The Extent of the Compasses from the Divisor {the Dividend} will reach (being apply'd the same way) from {Unity the Dividend} to the Quotient, in Whole Numbers and Decimal Fractions, if the Quotient be small; but if the Quotient exceed 20 or 30, you will hardly distinguish it nearer than to Whole Numbers only.

Example 1.

Let it be requir'd to divide 77 by 4. Here, the Extent of the Compasses from 4 to 1, will reach (the same way) from 77 to 19, 25; that is, $19\frac{1}{4}$ the Quotient sought.

Example 2.

Let it be requir'd to divide 240, by 15. Here, the Extent from 15 to 1, will reach from 240 to 16, the Quotient sought.

What has been said, together with the following Notes, being well read and practis'd, is sufficient to make this Rule plain to the meanest Capacity.

Note 1. It matters not where you take your *One*, whether at the Beginning, or Middle of the Line: Or if your Compasses be too short to reach backward to 1, at the Beginning or Middle of the Line; you may extend them forward to 1, at the Middle or End of the Line; provided that you know what Denomination to give to your Quotient: Which you may know by the following Note.

Note 2. The Number of Figures in the Quotient, is always so many as the Figures of the Dividend exceed the Figures of the Divisor; and one more when the two first Figures of the Dividend are greater than the two first Figures of the Quotient.

C H A P.

CHAP. IV.

Reduction by the Line.

Reduction is a Rule, that teaches how to bring a Number from one Denomination to another.

PROP. I.

To reduce any Greater Denominations into Lesser Denominations.

The ANALOGY.

As 1, to the Number of those Lesser Denominations contain'd in one of the Greater Denominations; so is the Number of those given Greater Denominations, to the Number of those Lesser Denominations equivalent thereunto.

Example.

In 20l. how many Shillings? Ans. 400. For, as 1: 20:: 20: 400. In the same manner 20 Peches, will be found to be 110 Yards. For as 1: 5:5: 20:: 110. And so of all other Denominations, whether they be Weight, Measure, Time, or the like.

PROP. II.

To reduce any Lesser Denominations into Greater Denominations.

This is but the Converse of the former Proposition; and therefore to solve it, this is

The ANALOGY.

As the Number of those Lesser Denominations that make one of those Greater Denominations, is to 1; so is the given Number of those Lesser Denominations, to the Number of those Greater Denominations equivalent thereunto.

Exam-

Example. II

In 400 Shillings, how many Pounds? Ans. 20. For, as 20 : 1 :: 400 : 20. Again, In 210 Yards, how many Perches? Ans. 20. For, as $5\frac{1}{2}$: 1 :: 110 : 20.

P R O P. III.

To reduce a Fraction from one Denominator to another,

The ANALOGY.

As the Given Denominator, is to his Numerator; so is the Intended Denominator, to his Numerator.

Example. What part of 20, is $\frac{1}{5}$? Ans. $\frac{1}{5}$. For, as 12 : 3 :: 20 : 5.

What part of 20, is $\frac{1}{5}$? Ans. $\frac{1}{5}$. For, as 12 : 3 :: 20 : 5.

P R O P. IV.

To reduce a Vulgar Fraction to a Decimal Fraction.

The ANALOGY.

As the Denominator of the Given Fraction, is to his Numerator; so is 1 (at the Middle or End of the Line), to the Decimal Fraction requir'd.

Example.

What is $\frac{1}{5}$, being reduc'd to a Decimal Fraction? Ans. .25. (i. e. 2 Primes, 5 Seconds.) For, as 12 : 9 :: 1 : .25.

P R O P. V.

To reduce Moneys, Weights, Measures, &c. into Decimals.

To do this, Observe, that the greatest Denominations of Moneys, Weights, &c. are commonly counted Integers: this known, the Decimal of any smaller Denominations are found by this

R U L E.

Consider what Vulgar Fraction your given Weights, Moneys, &c. are of an Integer; and by Prop. IV. reduce that Vulgar Fraction to a Decimal.

Ex-

Example.

What is the Decimal of $5s$? Ans. In consideration that Pounds are the greatest Denominations of Moneys; therefore one Pound is the Integer: then because $5s$ is $\frac{5}{20}$ of a Pound, I reduce $\frac{5}{20}$ (by Prop. IV.) into a Decimal, and it brings me .25, which is the Decimal (of a Pound) answering to $5s$.

By the same Method, the Decimal (of a Foot) answering to 9 Inches, will be found to be .75. And so of any other Moneys, Measures, Weights, Times, or the like.

PROP. VI.

To reduce a Decimal Fraction to a Vulgar Fraction.

This Proposition is but the Converse of the 4th. And therefore to solve it, this is

The RULE.

As 1 (at the Middle, or End of the Line), is to the Given Decimal; so is the Given Denominator, to his Numerator. So this Decimal .75 of a Gallon, will be found to be $\frac{6}{8}$, that is, 6 Pints.

PROP. VII.

To reduce a Decimal to his known Parts of Coyn, Weight, Measure, &c.

The RULE.

As 1 (at the Middle, or End of the Line), is to the Given Decimal; so is the Number of Parts that make an Integer, to the Number of those Parts equivalents to the Given Decimal.

Example.

What is the Value of this Decimal .125 of a Pound? Ans. 2s. 6d. For as $1 : .125 :: 20s. : 2.5s.$ i. e. 2s. and .5 of a Shilling. Then, as $1 : .5 :: 12d. : 6d.$

Thus .125 of a Foot, will be found to be 1 $\frac{1}{2}$ Inch. And so of any other Denominations.

PROP.

Ex.

P R O P. VIII.

To reduce a Whole Number to an Improper Fraction.

The ANALOGY.

As 1, to the Given Number; so is the Intended Denominator, to his Numerator.

So, if 2 were requir'd to be reduced to an Improper Fraction, whose Denominator should be 3; it would be found to be $\frac{6}{3}$. And so of any other Number.

P R O P. IX.

To reduce an Improper Fraction to its equivalent Whole, or Mixt Number.

The ANALOGY.

As the Denominator of the Given Fraction, is to his Numerator; so is 1, to his equivalent Whole, or Mixt Number.

Thus Working, you will find $\frac{6}{3}$ to be equal to 2, and $\frac{7}{3}$ to be equal to 2. $\frac{1}{3}$; i. e. $2\frac{1}{3}$. and so of others.

P R O P. X.

To reduce a Compound Fraction to a Simple One of the same Value.

The RULE.

If the Compound Fraction consist but of two Fractions; then say, As the Numerator of one of the Fractions, is to his Denominator; so is the Denominator of the other Fraction, to a new Denominator, to his former Numerator: Which new Fraction is equal to the Compound Fraction given.

Thus $\frac{1}{2}$ of $\frac{1}{3}$ will be found to be equal to $\frac{1}{6}$.

But if the Compound Fraction consist of three (or more) Fractions; then find a Simple Fraction equal to any two of them, as before; and say, As the Numerator

Numerator of this New Fraction, is to his Denominator; so is the Denominator of one of the rest of the Fractions, to a new Denominator to their Fraction; thereby making him equal to them three. Then, if the Compound Fraction consist of more Fractions, say (as before) As the Numerator of *this* New Fraction, is to his Denominator; so is the Denominator of one of the rest of the Fractions, to a new Denominator to that Fraction; thereby making him equal to (the Composition of) them four. Proceed in the same manner if there were more Fractions in the given Compound Fraction.

Example.

What is the Value of $\frac{1}{3}$ of $\frac{2}{4}$ of $\frac{3}{5}$ of $\frac{3}{8}$? Ans. $\frac{1}{108}$.
For, as $1:3::4:12$. then, as $2:12::6:36$. then, as $2:36::6:108$, the Denominator to 3, the last Numerator.

Here, Note. It is the best way to leave that Fraction till last, which has the least Numerator; for so the Simple Fraction will be in the lesser Terms. So, if (in this Example) we had left the $\frac{1}{3}$ till last, it would have been brought to $\frac{1}{36}$.

P R O P. XI.

To reduce a Fraction to its least Terms.

The R U L E.

Extend the Compasses from the Numerator of the Given Fraction, to his Denominator; then apply that Extent, the same way, from 1, 2, or 3, or what other Number you can find will make the other Foot of the Compasses rest in a Whole Number; for then the lesser of those two Numbers where the Compasses rest, is the Numerator, and the greater is the Denominator of the Fraction reduc'd to his least Terms.

Thus $\frac{1}{2}$ will be found equal to $\frac{1}{2}$, and $\frac{2}{3}$ will be found equal to $\frac{2}{3}$.

P R O P. XII.

To reduce Foreign Weights and Measures into English ones; and the contrary.

To solve this Proposition; take the following Tables, and the Rules following them.

1. A TABLE, carefully comparing Foreign Weights and Measures with the English.

Places Names.	Foot 1000 Parts	Pounds 100 Parts
London Foot.	1. 000	
London Pound Averd.	- - -	1. 00
France.		
Paris Royal Foot.	1. 068	0. 93
Lyon Ell.	3. 976	1. 09
Rologn Ell.	2. 076	0. 89
XVII Provinces.		
Amsterdam Foot.	0. 942	0. 93
Amsterdam Ell.	2. 269	
Antwerp Foot.	0. 946	0. 98
Antwerp Ell.	2. 273	
Brill Foot.	1. 103	
Dert Foot.	1. 184	
Leyden or Rynland Ell.	1. 033	0. 96
Leyden or Rynland Ell.	2. 260	
Lorain Foot.	0. 958	0. 98
Mecklin Foot.	0. 919	0. 98
Middleburg Foot.	0. 991	0. 98

Ger-

Places Names.	1000 Parts	Pounds 100 Parts
Germany.		
Strasburg Foot.	0. 920	0. 93
Bremen Foot.	0. 964	0. 94
Cologn Foot.	0. 954	0. 97
Frankford (on the Main) Foot.	0. 948	0. 93
Frankford (on the Main) Ell.	1. 825	
Hambrough Ell.	1. 905	0. 95
Leipsic Ell.	2. 260	1. 17
Lubeck Ell.	1. 903	
Nurembourg Foot.	1. 006	0. 94
Nurembourg Ell.	2. 227	
Bavaria Foot.	0. 954	
Vienna Foot.	1. 053	0. 83
Spain and Portugal.		
Spanish Palm, or Palm of Castile.	0. 751	0. 99
Spanish Vare, or Rod, 4 Palms.	3. 004	
Spanish Foot, $\frac{1}{3}$ of the Vare.	1. 001	
Lisbon Vare.	2. 750	1. 06
Gibraltar Vare.	2. 760	1. 03
Toledo Foot.	0. 899	1. 00
Toledo Vare.	2. 685	
Italy.		
Roman Foot on the { Cullatius	0. 967	1. 23
Monument of { Statilius.	0. 972	
Roman Palm for Building, whereof 10 make a Cauna. }	0. 732	
Bononia Foot.	1. 204	1. 27
Bononia Perch, (whereof 500 make a Mile.) }	12.040	
C 2		Floréncia

Places Names.	1000	Pounds
	Feet	100 Parts
Florence Brace or Ell.	1. 913	1. 23
Naples Palm.	2. 100	1. 43
Naples Cauna.	6. 880	
Genoa Palm.	0. 830	1. 42
Mantua Foot.	1. 569	1. 43
Milan Calamus.	6. 544	1. 40
Parma Cubit.	1. 866	1. 43
Venice Foot.	1. 162	1. 53
Other Places.		
Dantzick Foot.	0. 944	1. 19
Dantzick Ell.	1. 903	
Copenhagen Foot.	0. 965	0. 94
Prague Foot.	1. 026	1. 06
Turin Foot.	1. 962	
Riga Foot.	1. 831	
China Cubit.	1. 016	
Cairo Cubit.	1. 824	1. 61
Persian Arash.	3. 197	
Turkish Pike (at Constantinople)	3. 200	0. 86
the Greater.		
The Greek Foot.	1. 007	

2. A T A

2. A TABLE, comparing Foreign Measures with
the English, By HENRY PHILLIPS.

Places Names.	10000 Ells Parts
London Ell.	L. 0000
Amsterdam Ell.	O. 5900
Antwerp Ell.	O. 6000
Arras Ell.	O. 6060
Bridges Ell.	O. 6097
Nuremberg Ell.	O. 5747
Coten Ell.	O. 4807
Lisse Ell.	O. 6024
Mastricht Ell.	O. 6300
Frankford Ell.	O. 4792
Dantzick Ell.	O. 7228
Vienna Ell.	O. 6896
Paris Aulne.	L. 0526
Rouen Aulne.	O. 9708
Lyons Aulne.	O. 9826
Callais Aulne.	O. 6369
Venice Linnen Brace.	O. 5555
Venice Silk Brace.	O. 5102
Luques Brace.	O. 5000
Florence Brace.	O. 4901
Milan Brace.	O. 4347
Leghorn Brace.	O. 5000
Isles of Madera Brace.	O. 9681
Sevil Varc.	O. 7407
Lisbon Varc.	L. 0000
Castile Varc.	O. 7207
Andalusia Varc.	O. 7339
Granado Varc.	O. 7339
Genoa Palm.	O. 2079
Saragossa Cane.	L. 1881
Rome Cane.	L. 7857
Barcelona Cane.	L. 4035
Valentia Cane.	O. 8247

3. A TABLE, comparing Foreign Weights with
the English. By HENRY PHILIPS.

Places Names.	Pounds Parts
London Pound Averd.	1. 0000
Antwerp Pound.	0. 9615
Amsterdam Pound.	0. 9000
Abbeville Pound.	0. 9100
Ancona Pound.	1. 2820
Avignon Pound.	1. 1200
Bordeaux Pound.	0. 9100
Burgoin Pound.	0. 9100
Bologna Pound.	1. 2500
Bridges Pound.	0. 9800
Callabria Pound.	1. 3698
Callais Pound.	1. 0700
Constantinople Pound.	0. 8474
Diet Pound.	0. 9100
Dantzick Pound.	1. 1600
Ferrara Pound.	1. 3333
Florence Pound.	1. 2880
Flanders (all) Pound.	1. 0600
Geneva Pound.	0. 9345
Genoa Pound { Subtle.	1. 4084
	1. 4285
Hamburg Pound.	0. 9200
Holland Pound.	0. 9500
Luzerne Pound.	0. 8810
Lyons Pound { Common.	1. 0700
	0. 9800
	0. 9000
Leghorn Pound.	1. 3333
Milan Pound.	1. 4285
Mirandola Pound.	1. 3333
Norim-	

Places Names.	10000	
	Pounds	Parts
Norimberg Pound.	0.	8300
Naples pound.	1.	4084
Paris pound.	0.	8900
Prague pound.	0.	8300
Placentia pound.	1.	2882
Rochel pound.	1.	1200
Rome pound.	1.	2700
Rovan pound	{ Vicont.	
	{ Common Weight.	
Svil pound.	0.	8750
Thoulousa pound.	0.	9017
Turin pound.	1.	0800
Venice pound	{ Subtle.	
	{ Gros.	
Vienna pound.	1.	1200
	1.	2195
	1.	5625
	0.	9433
	0.	8130

The Use of the foregoing Tables is very easy. Thus,
 Say, As 1, is to the Number in the { 2d } Column
 (of the first Table) against the given Place; so is the
 given Number of { Measures } to the Number of
 { Feet } equivalent thereto.
 { Pounds Averd. }

Example 1.

Let it be requir'd to reduce 100 Paris Royal Feet into
 English. To do which I say, As 1 : 1.068 :: 100 : 106.8
 the Answer sought.

Example 2.

Let it be requir'd to reduce 100 Pound at Paris into
 English Pounds. Here I say, As 1 : 0.93 :: 100 : 93. the
 Answer.

So, by these two Examples it appears, That 100 *Paris* Royal Feet are equal to 106.8 Feet *English*; And 100 *Paris* Pounds are equal to 93 Pounds *Averd. English*. By the same Method you may reduce any other Foreign Weight or Measure into *English*. The thing is too plain to need another Example.

To reduce *English* Feet or Pounds into Foreign, is but the Converse of the former; and therefore to do it, this is

The R U L E.

Say, As the Number, in the $\left\{ \begin{smallmatrix} 2d \\ 3d \end{smallmatrix} \right\}$ Column (of the first Table) against the given Place, is to 1; so is the Number of *English* $\left\{ \begin{smallmatrix} \text{Feet} \\ \text{Pounds Averd.} \end{smallmatrix} \right\}$ to the Number of Foreign $\left\{ \begin{smallmatrix} \text{Measures} \\ \text{Pounds} \end{smallmatrix} \right\}$ equivalent thereto.

Example 1.

In 1068 *English* Feet, how many *Paris* Royal Feet?
Answer, 1000. For, as 1.068 : 1 :: 1068 : 1000.

Example 2.

In 93 Pound *Averd. English*, how many Pound at *Paris*? Answer, 100. For, As 0.93 : 1 :: 93 : 100.

The Use of the second and third Table is the very same as the first; only the second Table reduces Foreign Measures into Ells, and not into Feet.

CHAP. V.

Progression by the Line.

Progression is of two Kinds, Arithmetical and Geometrical.

An Arithmetical Progression is made by the continual Addition of any certain Number (as suppose, 1, or 2, or 3, &c.) to any given Number. Thus, 1, 2, 3, 4, and 2, 4, 6, 8, &c. 2, 5, 8, 11, &c. are Arithmetical Progressions.

A Geometrical Progression is made by the continual Multiplication (of any given Number) by any certain Number; as suppose by 2, or 3, or 4, &c. Thus, 2, 4, 8, 16, &c. 1, 3, 9, 27, &c. are Geometrical Progressions.

The Number by which any Progression is made, i. e. the Number continually added to the given Number, in an Arithmetical Progression, is called the *Excess*; or the Number by which the given Number is continually multiply'd, in a Geometrical Progression, is called the *Ratio*. This being premis'd, I come to shew how in

PROP. I.

To find the last Number or Term of any Arithmetical Progression.

The ANALOGY

As 1, is to a Number less by 1 than the Number of the Places in the given Progression; so is the Excess, to a fourth; to which fourth, add the first Number (or Term,) and their Sum is the last Number or Term sought.

Example.

What is the last Term of a Progression of six Places, whose first Term is 1, and Excess 9? Answer, 46. For, as $1:5::9:45$. To which, adding the first Term, it makes 46.

PROP.

PROP. II.

To find the Total Sum of any Arithmetical Progression.

The ANALOGY.

As 1, is to the Sum of the first and last Terms of the given Progression; so is half the Number of Places, to the Total Sum of the Progression, sought.

Or thus:

As 2, is to half the Sum of the first and last Terms; so is the Number of Places, to the Total Sum of the Progression, sought.

PROP. III.

To find the last Term or Number of any Geometrical Progression.

The RULE.

Extend the Compasses from 1, to the Excess; and with that Extent, let one Foot of the Compasses in the first Term, and turn them forward once less than the Number of Terms, and they will rest on the last Term, sought.

Example.

What is the last Term of the Geometrical Progression, whose first Term is 1, Ratio 2, and Number of Terms 6? Answer, 32. For (according to the Rule) the Extent from 1, to 2, (the Excess) will reach at six times turning, from 1 (the first Term) to 32, the last Term, sought.

PROP. IV.

To find the Total Sum of any Geometrical Progression.

The RULE.

As the Ratio less by 1, is to the last Term less by the first Term; so is the Ratio to a fourth. To which fourth,

fourth add the first Term, the Sum is the Total Sum of the Progression.

Example.

What is the Total Sum of a Geometrical Progression, whose first Term is 2, Ratio 3, and last Term 54? Answer, 80. For, as 2, (the Ratio less by 1), is to 52, (the last Term less by the first); so is 3 (the Ratio), to 78; (the fourth) to which adding 2 (the first Term) it makes 80, the Total Sum of the Progression, sought.

I might have added several other Propositions in Progression; but (those already exhibited being the most useful, and the others containing more of Curiosity than Use,) I have purposely omitted them.

C H A P. VI.

The Golden Rule Direct, by the Line.

(1.) **T**His Rule may well be call'd the *Golden Rule*, it being the most Excellent and Useful Rule of all the Rules in Arithmetick.

(2.) This Rule teaches the Resolution of any Analogy or Proportion; that is, it shews how, by having three Numbers given, to find a fourth that shall bear the same proportion to the third, that the second doth to the first. Now

(3.) For Working this Rule by the Line, this is

The RULE.

Say, As the first given Term, is to the second; so is the third, to the fourth, sought.

Or thus.

As the first given Term, is to the third; so is the second, to the fourth, sought.

Wehre-

Wherefore,

Always extend the Compasses from the first given Term or Number, to the second; and that Extent of the Compasses, apply'd the same way, upon the Line, shall reach from the third given Term or Number, to the fourth, requir'd.

Otherwise,

Extend the Compasses from the first Term, to the third; and that Extent, apply'd the same way, shall reach from the second, to the fourth, requir'd.

(5.) Either of these ways will effect the same thing: And 'tis necessary thus to vary the Method sometimes, to avoid opening the Compasses too wide; for when they are open'd to a very large Extent, you can neither take off any Distance so exactly, nor give so good an Estimate of any Parts requir'd, as you may do when they are open'd to a lesser Distance.

But all the Difficulty in this Rule, consists in the right Ordering of the Terms (or Numbers) in any Question propounded: For some Questions are naturally so intricate, or else they are so intricately stated, that 'tis no easy matter to know which is the first, second, or third Term. And therefore, that the Reader may not be at a loss in this Point, I shall here lay down this Plain and General

R U L E,

(6.) To place the Terms right in any Question of the Golden Rule.

In all Questions of the Golden Rule, there be three Terms propounded, viz. two of Supposition, and one of Demand; and that of Demand is always the third Term; and the Term of Supposition, of the same Denomination with the Term of Demand, must be the first Term; and the other Term of Supposition, the second Term.

(7.) When the Terms (in any Question in the Golden Rule) have been rank'd in their due Order, by the foregoing Rule; then the Question is to be solv'd by the fourth Rule of this Chapter.

(8.) If

(8.) If the Compasses be too short (as many times they are) to reach from the first Term, to the second; or from the first, to the third; then you must take several Denominations of Number upon the same Radius of the Line; (e. g. from Units to Tens, or Hundreds, &c.) and observe to give a right Denomination to the fourth Term. But, because in such a Case the Learner may be at a loss to know what Denomination (whether Units, or Tens, or Hundreds, &c.) to give to the fourth Term; I shall here lay down

A General RULE,

(9.) To work the Golden Rule, when the Compasses are too short to reach from the first Term, to the second or third.

In this Case, you must take the second or third Term upon the same Radius (of the Line) with the first Term. e. g. If the Question were, If 2 give 30, what shall 50 give? Here the Distance from 2 (in the first Radius) to 30, or 50 (in the second Radius) is too great for most ordinary Compasses; Wherefore, I extend the Compasses from 2 to 3 (in the same Radius) accounting 3 for 30; and that Extent apply'd the same way from 50, (in the other Radius) reaches to 75. But now, to know what Denomination to give to this 75, observe this

General RULE,

(10.) To know how many Figures there should be in the fourth Term of any Proportion.

First find (by Note 2d in Multiplication) how many Figures would be in the Product of the second and third Terms, which account as the Number of Figures in a Dividend; and then reckon the first Term for a Divisor, and find (by Note 2d in Division) how many Figures would be in the Quotient; for so many there will be in the fourth Term.

So, in the last Example, in the Product of 30 and 50 (the second and third Terms) there will be found

D

(by

(by Note 2d in Multiplication) 4 places of Figures; which being accounted for the Number of Figures in a Dividend, and reckoning 2 (the first Term) for a Divisor; there will be found (by Note 2d in Division) to be 3 places of Figures in the Quotient; and so many there will be in the fourth Term. Therefore, that 75, found to be the fourth, must be accounted 750. This is very plain and easy; but 'tis also very tedious: Therefore

(11.) In this Case of the Compasses being too short, I use to work thus:

I take the second or third Term upon the same Radius (of the Line) that I take the first Term on, (according to the Ninth Rule of this Chapter) and call it by the same Denomination with the first Term; (whether it be Units, or Tens, or Hundreds, &c.) But this will be better illustrated by an Example: Wherefore I take that Example in the Ninth Rule; viz. If 2, give 30; what shall 50 give? Here I take 30, at 3, in the same Radius with 2, (the first Term) and call it by the same Denomination with the first Term, viz. Units. Then I reason thus; If 2 give 3, 50 will give 75, (by the third and fourth Rule of this Chapter.) Then I reason again, thus: But my second Term is 30, which is 1 Denomination higher than I suppos'd it; (for 30, is Tens; and 3, which I took for 30, is but Units;) and therefore my fourth Term must be 1 Denomination higher than I found it by the former Supposition; (i. e. it must be Hundreds.) Therefore the fourth Term will be 750. Which is the Answer to the Question.

This way is quicker than the former; and there is no difficulty in it, if you do but observe to denominate your fourth Term so many Denominations higher or lower, than you found it by your suppositious Analogy, as your second Term is higher or lower than you had suppos'd it.

(12.) I would have the Learner often practice (so as to be

be very perfect in) the Golden Rule; for he'll find that most of the Rules in this Book, are laid down Analogically; and so to be solv'd by this Rule.

(13.) Therefore, this Rule being of such excellent Use in all Parts of the Mathematicks, (as will appear from the following Pages of this Book,) I shall need here, only to propose a few Questions, whereby to shew its Excellency in resolving Questions purely Arithmetical.

Qu. 1. If 100 *l.* yield 4 *l.* Interest for a Year; what shall 75 *l.* yield? Answer, 3 *l.* For, as 100:4::75:3.

Qu. 2. If one Acre of Land be worth 15 *s.* a year, what is 40 Acres (of the like Land) worth by the year? Answer, 600 *s.* which (by Prop. 2. in Reduction) makes 30 *l.*

Qu. 3. Cloth at 5 *s.* the Yard; what comes 36 Yards to? Answer, 180 *s.* equal to 9 *l.* For, as 1:5::36:180. or, as 1::25 *l.*::36:9 *l.*

Qu. 4. Nails at 5 *d.* the Hundred; how many for 3 *d.*? Answer, 60. For, as 5 *d.* 100::3 *d.* : 60.

Qu. 5. If a Servant have 5 *l.* a Year; what is that a Day? Answer, 3. 29 *d.* For, as 365 (the Days in a Year): 1200 (the Pence in 5 *l.*): 1 (Day): 3. 29 *d.* something better than 3 *d.* 1 *s.*

Qu. 6. If 108 *l.* in 12 Months, gain 6 *l.* Interest; much will it gain in 7 Months? Answer, 3 *l.* 10 *s.* Answer, 3 *l.* 10 *s.* For, as 12:6::7:3. 5 *l.* equal to 3 *l.* 10 *s.*

Almost an infinite Variety of Examples might be added; but for Brevity sake, let these serve as a taste of the rest.

(14.) The Effects of the Golden Rule Direct, are

1. Its first and general Effect, is to find a fourth Proportional, consisting of two equal Reasons. The Rule for this is laid down before, in the third and fourth Rules of this Chapter.

2. The second Effect is, by the Price or Value of any one thing, to find the Price or Value of many things, of

the like Kind and Goodness; by saying, As one thing, is to its Price; so is any Number of Things, to their Price.

3. The third Effect is, by the Price or Value of many things, to find the Price of one; by saying, As any Number of Things, is to their Price; so is one, to its Price.

4. The fourth Effect is, by the Price of any Number of Things, to find the Price of any other Number of Things, of the like kind; by saying, As any Number of Things, is to their Price; so is any other Number of Things, to their Price.

5. The fifth Effect is, the reducing of Moneys, Weights, or Measures, from one Denomination to another. The Rules for which are already taught in the Chapter of Reduction.

So let this suffice for the Golden Rule Direct.

CHAP. VII.

The Golden Rule Reverse, by the Line.

(1.) THE Golden Rule Reverse, is that by which, from three Numbers given, we find a fourth in a reciprocal Proportion, inverted to the Proportion given.

(2.) For Working of this Rule, this is the

ANALOGY.

As the third Term, is to the $\left\{ \begin{smallmatrix} 1st, \\ 2d, \end{smallmatrix} \right\}$ so is the $\left\{ \begin{smallmatrix} 2d \\ 1st \end{smallmatrix} \right\}$ to the fourth.

Wherefore,

Always extend the Compasses from the first Term, to the $\left\{ \begin{smallmatrix} 1st, \\ 2d, \end{smallmatrix} \right\}$ the same Extent shall reach (being apply'd the same way) from the $\left\{ \begin{smallmatrix} 2d, \\ 1st, \end{smallmatrix} \right\}$ to the 4th.

Or,

Or, Extend the Compasses from the first Term to the third, the same Extent (being apply'd the contrary way) shall reach from the second Term, to the fourth.

(4.) This Rule being but of small Use in other Parts of the Mathematicks, I shall here exemplify its Usefulness in Arithmetick.

Qu. 1. If 8 Men do a Piece of Work in 12 Days, in how many Days will 16 Men do the same? *Ans.* In 6 Days.

Qu. 2. If 8 Men do a Piece of Work in 12 Days, how many Men must there be to do the same in 6 Days? *Ans.* 16. This Question may prove the foregoing.

Qu. 3. If (according to the Standard, Statute, or Law of England) the Penny-Loaf

{ White, }
{ Wheaten, } must
{ Household, }

weigh, when Wheat is at Five Shillings the Bushel,

{ 7 oz. 7 d. w. }
{ 11. oz. } Troy, or { 8 oz. }
{ 1 lb. 2 oz. 14 d. w. } { 12 oz. }
{ 16 oz. or 1 lb. }

Averd. How much must the Penny-Loaf (White, Wheaten, or Household,) weigh, when Wheat is at

2s. 6d. the Bushel? *Ans.* The Penny-Loaf

{ White, }
{ Wheaten, }
{ Household, }

must Weigh by Troy-Weight { 1 lb. 2 oz. }
{ 1 lb. 10 oz. } and
{ 2 lb. 5 oz. 8 d. w. }

by Averd. Weight { 1 lb. }
{ 1 lb. 8 oz. }
{ 2 lb. }

Qu. 4. How many Pieces of Money or Merchandise, valu'd at 20s. per Piece, are to be given or receiv'd

for 240 Pieces, valu'd at 12s. per Piece? *Ans.* 144.

For, As 12s: 240 Pieces:: 20s: 144 Pieces.

Qu. 5. How many Yards of Canvas that is Ell wide, will suffice to Line 20 Yards of Say that is three Quar-

ters (of a Yard) wide? *Ans.* 12 Yards. For 3:20:: 9:12 reciprocally.

Qu. 6. How many Foot of Matting, of 2 Foot wide, will cover a Floor 20 Foot wide, and 24 Foot long? *Ans.* 240 Foot. For, As 20 : 21 :: 2 : 240.

Qu. 7. If (according to the Statute) a Right-Angl'd Paralelogram of 40 Perches in length, and 4 in breadth, make an Acre of Land; how much in length must there be, when the breadth is 16 Perches? *Ans.* 10 Perches. For, As 4 : 40 :: 16 : 10.

Qu. 8. If 12 Inches in breadth, and 12 in length, make a Superficial Foot; how much must there go in length to make a Foot, when the breadth is but 3 Inches? *Ans.* 48 Inches. For, As 12 : 12 :: 3 : 48.

Qu. 9. I lent my Friend 600 *l.* for 8 Months; how much must he lend me again for 12 Month, to recompence my Curtesy? *Ans.* 400 *l.* For, As 8 M : 600 *l.* :: 12 M : 400 *l.*

Qu. 10. I lent my Friend 600 *l.* for 8 Months; for how long time must he lend me 400 *l.* to recompence my Curtesy? *Ans.* For 12 Months. For, As 600 *l.* 8 M. 400 *l.* 12 Months.

Qu. 11. If for 24 *s.* I have 1200 lb. Weight carry'd 36 Miles; how many Miles shall 1800 lb. Weight be carry'd for the same Money? *Ans.* 24 Miles. For, As 1200 : 36 :: 1800 : 24.

Qu. 12. If for 24 *s.* I have 1200 *l.* Weight carry'd 36 Miles; how many Pound Weight shall I have carry'd 24 Miles for the same Money? *Ans.* 1800 lb. Weight. For, As 36 : 1200 :: 24 : 1800.

Qu. 13. A Colonel is besieg'd in a Town, in which are 1200 Souldiers, with Provision for 3 Months; how many of his Souldiers must he dismiss, that his Victuals may last the rest of his Men 8 Months? *Ans.* He must keep 450 Men, and dismiss the rest. For, As 3 : 1200 :: 8 : 450.

Qu. 14. If a Field will graze 21 Horses, for 6 Weeks; how many will he graze for 7 Week? *Ans.* 18. For, As 6 : 21 :: 7 : 18.

Qu. 15. If a Field will graze 21 Horses, for 6 Weeks; for

for how long will it graze 11 Horses? Ans. For 7 Weeks.
For, As 21:6::18:7.

Qu. 16. How many Groats are equivalent to 24 Sixpences? Ans. 36. For, As 6d. 24::4d. 36.

Qu. 17. At 5d. a piece; what comes 48 Things to? Ans. 20s. For, As 5d:48::12d (equal to 1s.) :20 (12d's or) Shillings.

Qu. 18. Things at 5d. a piece; how many shall I buy for 20s.? Ans. 48. For, As 12d: 20 (12d's)::5d: 48 (5d's).

I forbear to mention any more Examples in this Rule, those being sufficient to shew the Excellency thereof; but the Ingenious Reader may propose an almost Infinity (if I may so speak) of other useful Questions, which may be solv'd by this Rule.

(5.) The Effects of the Golden Rule Reverse, are

1. The first and general Effect of the Golden Rule Reverse, is contain'd in the Definition of the same; that is, to find a fourth Term, in a reciprocal Proportion, inverted to the Proportion given. The Rule for this is laid down before, in the second and third Rule of this Chapter.

2. The second Effect is, by the Prizes, or Values of two several Pieces of Money, or Merchandise; To find how many pieces of one Price, are equivalent to a given Number of the other. And consequently, to $\left\{ \begin{array}{l} \text{reduce} \\ \text{exchange} \end{array} \right\}$

one sort of $\left\{ \begin{array}{l} \text{Money into} \\ \text{Merchandise for} \end{array} \right\}$ another. This is exemplify'd in Qu. 4. 16, 17, and 18.

3. The third Effect is, by two differing Prizes of Wheat, and the Weight of a Loaf of Bread, made answerable to one of the Prizes; To find the Weight of the same Loaf, made answerable to the other Price; *Et à contra.* This is exemplify'd in Qu. 3.

4. The fourth Effect is, by having the length and breadth of one Paralelogram, and the length or breadth of another; To find the breadth or length of the other, that

that the later Paralelogram may be equal to the former; this has been exemplify'd in Qu. 3, 6, 7, and 8.

5. The fifth Effect is, by a Double Time, and a Capital Sum of Money borrow'd or lent; To find out another Capital Sum answerable to one of the given Times. Or otherwise, by two Capital Sums, and a Time answerable to one of 'em; To find a Time answerable to the other Capital Sum. This was exemplify'd in Qu. 9, and 10.

6. The sixth Effect is, by two differing Weights of Carriage, and the Distance one of 'em is carried, for a certain Price; To find the Distance the other Weight must be carry'd, for the same Price. Or otherwise. By two Distances, and a Weight carry'd one of the Distances for a certain Price; To find what Weight must be carry'd the other Distance, for the same Price. This was exemplify'd Qu. 11, and 12.

7. The seventh Effect is, by a double Number of Workmen, and the Time that one of them is doing a piece of Work; To find how long the other Number of Workmen will be doing the same piece of Work. Or otherwise. By a double Time, wherein a piece of Work was done, and the Number of Workmen doing it in one of those Times; To find the Number of Men to do it in the other Time. This was exemplify'd in Qu. 1, and 2.

The eighth Effect is, by a double price of Provision, and the Number of Creatures nourish'd for a certain time, with a certain quantity of it, bought at one of the prizes: To find how many of the same kind of Creatures, may be nourish'd, for the same time, with the same Value of Provisions, bought at the other price.

The ninth Effect is, by a certain quantity of Provisions, and the Number of Creatures it will nourish for a certain time: To find how many it will nourish for another certain time. Or otherwise. By a certain quantity of Provisions, and the Number of Creatures it will nourish for a certain time; to find how long the same quantity

quantity of Provisions will nourish another Number of the same kind of Creatures. This was exemplify'd in Q. 15, and 16.

Many more are the *Effects* of this Rule, which any one may easily discover; and therefore I purposely pretermitt 'em.

(6.) To know when a Question is to be resolv'd by the Direct, and when by the Reverse Rule.

The RULE.

Place the Terms in their due Order, (by Rule 6. of Chap. 6.) and then if Reason shew, that the { greater, } the third Term is, the { greater, } the 4th Term must be; then the Question is in the Direct Rule: But if Reason shew, that the { greater, } the third Term is, the { lesser, } the fourth Term must be; then the Question is in the Reverse Rule.

(7.) To work the Golden Rule without respect to Direct, or Reverse.

The RULE.

First place the Terms in their due Order, by Rule 6. of Chap. 6. And then, if Reason shew, that the fourth Term sought ought to be { greater } than the Term of Demand; Then say, As the { lesser } Term of Supposition, is to the { greater, } so is the Term of Demand, to the fourth Term sought.

C H A P. VIII.

The Double Rule of Three, or Golden Rule compos'd of Five Numbers, by the Line.

(1.) **T**His Rule has its Name, from its having Five Numbers given, to find a sixth in proportion thereunto: And is resolv'd by two single Rules of Three. But, before you can work this Rule, you must know how

(2.) To dispose the given Terms (or Numbers) in a due Order and Place, preparative for Resolution.

The R U L E.

In all Questions in this Rule, there are five Terms propounded; viz. Three Terms of Supposition, and two of Demand. Of the three Terms of Supposition, let that which has the same Denomination with the Term requir'd, be placed in the second place, and place the other two Terms of Supposition one above another in the first place; and then place the two Terms of Demand one above another in the third place. Only observe to place Numbers of like Denomination in the same Rank; as two in the upper, and two in the lower Rank. Then

(3.) To Resolve any Question in the Double Rule of Three, or Golden Rule compos'd; this is

The R U L E.

Having plac'd the given Terms (or Numbers) in their due Order, (by the second Rule of this Chapter;) then say, As the first Term in the upper Rank, is to the second; so is the third in the same Rank, to a fourth. Again, As the first Term in the lower Rank, is to the fourth last found; so is the other Term in the lower Rank, to the Term requir'd.

Note, Before you work the two Analogies, you must before find (by Rule 6. of Chap. 7.) whether they are

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are to be wrought by the Direct or Reverse Rule; and accordingly work them.

(4.) Some Questions exemplifying the Usefulness of this Rule.

Qu. 1. If 100*l.* Principal, in 12 Months, gain 6*l.* Interest; What will 75*l.* Principal, gain in 9 Months?
Ans. 3*l.* 7*s.* 6*d.*

For the Terms being placed in Order by the second Rule, will stand thus; $\left\{ \begin{array}{l} 100\text{ }l.: 6\text{ }l.: 12\text{ }M: \\ 75\text{ }l.: 9\text{ }M: \end{array} \right\}$ Then I say,

As 100*l.* 6*l.* :: 75*l.* 4*l.* 5*s.* (equal to 4*l.* 10*s.*) Again,
As 12*M.* 4*l.* 5*s.* :: 9*M.* 3*l.* 37*s.* equal to 3*l.* 7*s.* 6*d.*

Qu. 2. If 100*l.* Principal, in 12 Months, gain 6*l.* In how long time will 75*l.* gain 3*l.* 7*s.* 6*d.*?
Ans. in 9 Months.

Qu. 3. If 100*l.* Principal, in 12 Months, gain 6*l.* What Principal will gain 3*l.* 7*s.* 6*s.* in 9 Months?
Ans. 75*l.* These three Questions mutually prove one another.

Qu. 4. If the Carriage of 100*lb.* 30 Miles, cost 12*d.* What will the Carriage of 500*lb.* cost, being carried 100 Miles? Ans. 200*d.* equal to 16*s.* 8*d.*

Qu. $\left\{ \begin{array}{l} 5. \\ 6. \\ 7. \end{array} \right\}$ If a Regiment of 936 Souldiers, eat 351

Quarters of Wheat, in 168 Days; How many Quarters will suffice 11232 Souldiers for 56 Days? Or, how many Souldiers will 1404 Quarters suffice for 56 Days? Or, how many Days will 1404 Quarters suffice 11232 Souldiers? Ans. To the first, 1404 Quarters. To the second, 11232 Souldiers. And to the third, 56 Days.

Qu. $\left\{ \begin{array}{l} 8. \\ 9. \\ 10. \end{array} \right\}$ If 8 Men, in 7 Days, Mow 40 Acres;

How many Acres will 24 Men mow in 28 Days? Or, in how many Days will 480 Acres be mow'd by 24 Men? Or, how many Men must there be to mow 480 Acres in 28 Days? Ans. To the first, 480 Acres. To the

the second, 28 Days. And to the third, 24 Mor.

Qu. 11. If 40 s. is the Wages of 8 Men, for 5 Days; What shall be the Wages of 32 Men, for 24 Days? *Ans.* 768 s. equal to 38 l. 8 s.

Qu. 12. If 14 Horses eat 56 Bushels of Provender, in 16 Days; How many Bushels will 20 Horses eat in 24 Days? *Ans.* 120 Bushels.

Qu. 13. If 8 Cannons, in one Day, spend 48 Barrels of Powder; How many Barrels will 24 Cannons spend in 12 Days? *Ans.* 1728 Barrels.

Qu. 14. If in a Family, consisting of 7 Persons, there are drunk out 36 Gallons of Beer in 12 Days; How many Gallons will there be drunk out in 8 Days, by another Family, consisting of 14 Persons? *Ans.* 48 Gallons.

Qu. 15. If, when the Bushel of Wheat costs 3 s. 4 d. the Penny-Loaf weighs 12 Ounces; What shall a Loaf worth 9 d. weigh, when Wheat is at 10 s. the Bushel? *Ans.* 36 Ounces.

Qu. 16. A Man put out 75 l. to receive Interest for the same, and when it had continu'd 9 Months, he receiv'd (for Principal and Interest) 78 l. 7 s. 6 d. Now, at what rate *per Cent. per Annum*, did he receive Interest? *Ans.* at 6 l. *per Cent. per Annum*.

Qu. 17. If a Captain set 300 Pioneers to work, and they, in 8 Hours, cast a Trench of 200 Rods; How many Pioneers will be able, with a like Trench, in 3 Hours, to intrench a Camp 3400 Rods? *Ans.* 13600 Pioneers.

CHAP. IX.

The Rule of Fellowship, by the Line.

(1.) **T**HE Rule of Fellowship is when two (or more) Men trade together, with one Common Stock; and when they have gain'd, or lost, it shews how to find each Man's particular Gain, or Loss.

(2.) The

(2.) The Rule of Fellowship is of two Kinds: viz. either without Time, or with Time.

(3.) For Fellowship without Time, this is

The RULE.

As the whole Stock, is to the whole $\left\{ \begin{array}{l} \text{Gain} \\ \text{Loss} \end{array} \right\}$ so is each Man's particular Stock, to his particular $\left\{ \begin{array}{l} \text{Gain} \\ \text{Loss} \end{array} \right\}$.

Example.

Four Merchants, A. B. C. D. trading together, made a Common Stock of Money; to the Value of 240 £ . Whereof A. put into the Stock 30 £ . B. 50 £ . C. 60 £ . and D. 100 £ . This Stock they occupy'd till it was increas'd to 3000 £ . Now, What should each Man receive at the parting of the Money? Ans. A. must receive 375 £ . B. 625 £ . C. 750 £ . and D. 1250 £ . For, As to 240, is to 3000; so is 30 £ to 375 £ . and so is 50 £ to 625 £ . and so is 60 £ to 750 £ . and so is 100 £ to 1250 £ .

(4.) The same Rule serveth in a Case of Bankrupt, by saying,

As the whole Sum of the Bankrupt's Debts, is to the whole Value of his Goods; so is each Man's particular Debt, to his Share of the Goods; that each Man may bear his proportionable Part in the Loss.

Example.

If the Bankrupt's Goods be valu'd at 500 £ . and he be in debt 1000 £ . to 4 Men, A. B. C. D. of which 100 £ . is due to A. 200 £ . to B. 300 £ . to C. and 400 £ . to D. Now, what ought to be each Man's Quota or Share of the Goods? Ans. A. must have 50 £ . B. 100 £ . C. 150 £ . and D. 200 £ . So that you see, in this Example, each Man loseth $\frac{1}{2}$ his Debt.

(5.) A Case of Bankrupt may also be solved by this

RULE.

As the whole Sum of the Bankrupt's Debts, is to its
B Excess

Excess above the whole Value of his Goods, (which Excess is the whole Loss that all his Creditors must sustain;) so is each Man's particular Debt, to the Loss that he must sustain. This needs no Example.

(6.) Fellowship with Time, is wrought by this

RULE.

Multiply each Man's particular Stock by his Time;
then say, As the Sum of these Products, is to the
whole { Gain; }
 { Loss; } so is each Man's particular Product,
to his particular { Gain.
 { Loss.

Example.

Four Merchants, A. B. C. D. made a Common Stock, which at the Year's end was increas'd to 35145*l*. Of this Stock, A. laid in 669*l*. which he took again at 10 Months end; B. laid in 810*l*. for 8 Months; C. laid in 900*l*. for 7 Months; and D. laid in 1040*l*. for 12 Months. Now, what is each Man's Portion of the Gains? Ans. A. must have 7359*l*. B. 7128*l*. C. 6920*l*. and D. 13728*l*.

CHAP. X.

The Rules of Practice, by the Line.

(1.) **T**HE Rules of Practice is but a Compendium of the Golden Rule Direct, when the first Term is one. Its Use is

(2.) By knowing the Price of one thing, to find the Price of any Number of things at that Rate.

The RULE

Say, As one thing, is to its Price (reduc'd to its least Denomination;) so is any Number of Things, to their Price.

Example.

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Example.

Cloth at 2 s. 2 d. the Yard; What comes 1000 Yards to? Ans. 108 l. 6 s. 8 d. For, As 1: 26 d.: 1000: 2600 d. equal to 108 l. 6 s. 8 d.

CHAP. XI.

The Extraction of the square Root, by the Line.

THE Square Root of a Number is a Number which being multiply'd by it-self, produceth the given Number: Or, the Square Root of a Number may be defin'd, to be, a mean Proportional between the given Number and Unity. To find it, this is

The RULE.

Divide (with your Compasses) the Space (on the Line) between 1 and the given Number into 2 equal Parts; and that middle Point where the Compasses rest, shews the Square Root sought.

Example.

Let it be requir'd to extract the Square Root of 36. To do which, I divide the Distance (on the Line) between 36, and 1, into 2 equal Parts, and the Compasses rest upon 6. Which is the Square Root sought.

And thus the Square Root of 81, will be found to be 9; of 144, 12; of 256, 16; and so of other Numbers, as in this TABLE.

Root	Square	Root	Square	Root	Square
1	1	8	64	15	225
2	4	9	81	16	256
3	9	10	100	17	289
4	16	11	121	18	324
5	25	12	144	19	361
6	36	13	169	20	400
7	49	14	196	21	441

Note 1. If the Number of Figures (in the given Number) be odd; it will be best to place Unity, at 1 in the middle of the Line; for so the Root and Square will both fall forwards towards 10 at the end of the Line. But

Note 2. If the Number of Figures be even, it will then be best to place your Unity, at 10 at the end of the Line; and so the Root and Square will both fall backwards, towards the middle of the Line.

Note 3. Half the Number of Figures (in the given Number) if even; or the greater half, if odd, is equal to the Number of Figures in the Square Root.

CH A P. XII.

The Extraction of the Cube-Root, by the Line.

THE Cube Root of a Number, is a Number, which being multiply'd twice into itself, produceth the given Number. Thus the Cube-Root of 64 is 4: For 4 multiply'd by 4 is 16; and that (again) multiply'd by 4, produceth 64.

Or, The Cube-Root of a Number, may be defin'd, to be, the first of 2 mean Proportionals, between the given Number and Unity. To find it, this is

The RULE.

Divide (with your Compasses) the Space (on the Line) between 1 and the given Number, into 3 equal Parts; and where the first Part from 1, endeth, that Point (of the Line) exhibiteth the Cube-Root sought.

Example.

What is the Cube-Root of 216? Ans 6. For the Space between 1, and 216, being divided into 3 equal Parts, the Compasses will first rest on 6, the Root sought.

And

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And thus the Cube-Root of 729, will be found to be 9; of 1728, 12; and so of other Numbers, as in this

T A B L E.

Root	Cube	Root	Cube	Root	Cube
1	1	8	512	15	3375
2	8	9	729	16	4096
3	27	10	1000	17	4913
4	64	11	1331	18	5832
5	125	12	1728	19	6859
6	216	13	2197	20	8000
7	343	14	2744	21	9261

Note 1. If you add 2 to the Number of Figures in the given Number, and divide that Sum by 3, the Quotient will shew the Number of Figures in the Root.

Note 2. If (working according to the first Note) there do nothing remain; it will be best to place Unity at the 1 in the middle of the Line; and so the Root, the Square, and the Cube, will fall forwards towards 10, at the End.

But if 1 remain; then Unity may be placed at either End, viz. at 1 at the Beginning, or at 10 at the End; and so the Cube will be 1 Radius from the Unity, either forward, or backward.

But if 2 remain, then 10 at the End of the Line, must be Unity; and the Root, the Square, and the Cube, will always be in the same Radius of the Line; that is, between 10 at the End, and 1 in the Middle.

C H A P. XIII.

Interest and Annuities, by the Line.

(1.) **W**hen one Man lends Money to another for a time, upon condition, that he pay him so much *per Cent*, for the Use of it; such Money paid for the Use of it, is call'd the *Use, Loan, or Interest*; and the Money lent is call'd the *Principal*; and so much as is allow'd for the Use of 100 *l.* for a Year, is call'd the *Rate*. If at the Year's end, the Principal be not paid, and the Interest do not become a part of the Principal; then it is call'd *Simple Interest*: but if at each Year's end, the Interest become a part of the Principal; then it is call'd *Compound Interest*.

(2.) To find the Simple Interest of any Sum of Money, at any Rate, for any given Time.

The RULE.

As 100 *l.* is to the Rate; so is the Principal, to its Interest for one Year. Then for any other time say,

As $\left. \begin{array}{l} 100 \\ 365 \end{array} \right\}$ is to the given Time, in $\left. \begin{array}{l} \text{Years;} \\ \text{Months;} \\ \text{Days;} \end{array} \right\}$

so is the Interest for one Year, to its Interest for the given Time, in Years, Months, or Days.

(3.) If you would have the Answer exhibit the Amount or Aggregate of the Principal and Interest: then say,

As 100 *l.* is to 100 *l.* added to the Rate; so is the Principal, to the Amount of the Principal and Rate for one Year. Then for any other Time say,

As $\left. \begin{array}{l} 100 \\ 12 \\ 365 \end{array} \right\}$ is to the given Time, in $\left. \begin{array}{l} \text{Years;} \\ \text{Months;} \\ \text{Days;} \end{array} \right\}$

so is the Amount for one Year, to the Amount for the given Time, in Years, Months, or Days,

Example.

What is the Interest of 1000 *l.* for five Years, at 5 *per Cent. per Annum*? Ans. 250 *l.* (by the second Rule of this Chapter.) Which added to the Principal, gives 1250 *l.* for the whole Amount; the same that would have been found by this third Rule. So that you see, if 1000 *l.* be put out to Use for five Years, at the Rate of 5 *l. per Cent. per Annum*, it will be increas'd to 1250 *l.*

(4.) Rebate or Discount is when Money is due at the End of a certain Time; and the Debtor agrees with his Creditor, to pay him Ready-Money, if he will allow him so much (as they agree for) *per Cent. per Annum*, (in consideration of his receiving his Money before it be due.) I say, this Allowance is call'd Rebate or Discount; and the Creditor must receive so much Ready-Money, as being put out to Use, (at the Rate of Discount agreed on, and till the Time it was due,) it may amount to the just Sum that would be then due. Now,

(5.) To find the present Worth of any Sum of Money, due at the End of any Time to come, allowing Discount or Rebate at any Rate (propos'd) Simple Interest; this is

The RULE.

First find (by Rule 2d of this Chapter) the Interest of 100 *l.* at the Rate, and for the Time, the Rebate is to be allow'd: then say,

As 100 *l.* added to its Interest (now found), is to 100 *l.* so is the given Sum to its present Worth.

Or otherwise.

First find (by Rule 3d) the Amount of 100 *l.* at the Rate, and for the Time, the Rebate is to be allow'd: then say,

As the Increase or Amount of 100 *l.* (now found), is to 100 *l.* so is the given Sum, to its present Worth.

Exam-

Example.

What present Money will satisfy a Debt of 240 *l.* due at the End of 4 Years yet to come, Discount or Rebate being allow'd, at the Rate of 5 *l.* per Cent. per Annum? *Ans.* 200 *l.*

(6.) To find the Increase or Amount of any Annuity, or Yearly Rent or Income, in any Time; allowing Simple Interest (at any Rate per Cent. per Annum) for each Yearly Payment, from the Time it first became due.

The RULE.

Find (by Rule 2d) the Interest (for one Year, and at the Rate propos'd) of one Year's Payment; and multiply it by the given Number of Years, and multiply that Product by $\frac{1}{2}$ the Number of Years lessen'd by 1, (or multiply $\frac{1}{2}$ that Product by the Number of Years lessen'd by 1.) this last Product is the Sum of the Interest of the Annuities; which added to the Sum of the Annuities, is equal to the total Amount or Increase of the Annuities.

Example.

There is an Annuity or Yearly Rent, or Income of 200 *l.* per Annum, forborn to the End of 6 Years; Now, how much is due at the End of the said Time, allowing Simple Interest, at the Rate of 6 *l.* per Cent. per Annum? *Ans.* 1380 *l.* For the Sum of the Annuities is 1200 *l.* and the Sum of the Interest is 180 *l.* there being no Interest allow'd (in this Case) for the last Annuity, (or Year's Rent) that being paid as soon as it becomes due.

(7.) To find the present Worth of an Annuity or Yearly Rent, allowing Discount or Rebate, (at any Rate per Cent. per Annum) Simple Interest.

The RULE.

Find (by Rule 6. the Increase or Amount of the propos'd Annuity, for the propos'd Time, and at the propos'd

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posed Rate. Then (by Rule 5.) find the present Worth of this Sum (thus found) due at the End of the proposed Time; and that shall be the present Worth of the Annuity.

Example.

There is a Lease of a House or Land, worth 12 *l.* per Annum, and there is 16 Years of the Lease yet to come; which Lease a Man would buy, if he could lay out his Money so as to gain 10 *l.* per Cent. The Question is, What is this Lease worth? Ans. 129 $\frac{1}{3}$ *l.*

(8.) Compound Interest, or Interest upon Interest. What it is, has been shew'd above, in Number 1 of this Chapter. Now,

To find what any Sum of Money will be increas'd to, (being put out to Interest) in any Number of Years, and at any Rate per Cent. reckoning Compound Interest, or Interest upon Interest; this is

The RULE.

Extend the Compasses from 100 *l.* to the Increase of 100 *l.* in one Year; that Extent of the Compasses being turn'd (from the Sum propos'd) the same way, as many times as there be Years propos'd, shall at last lay at the Sum of the Principal and Interest requir'd.

Example.

What shall 143 *l.* 10 *s.* amount to in 30 Years, at the Rate of 6 *l.* per Cent. Compound Interest? Ans. 257 *l.* 10 *s.*

(9.) To find the present Worth of any Sum of Money, due at any Time to come, allowing Discount or Rebate, at any Rate per Cent. Compound Interest.

The RULE.

Extend the Compasses from the Increase of 100 *l.* in one Year, to 100 *l.* that Extent of the Compasses being turn'd (from the Sum propos'd) the same way, as many times as there be Years propos'd, shall at last lay at the present Worth sought.

Example.

Example.

What is the present Worth of 257*l.* due at the End of 10 Years, allowing Discount or Rebate at 6*l.* per Cent. Compound Interest? Ans. 143*l.* 10*s.*

(10.) To find the Increase or Amount of an Annuity or Yearly Rent, in any Number of Years, allowing Compound Interest, at any Rate per Cent.

The RULE.

First find what Principal will (at the propos'd Rate) in one Year, gain the Annual Rent propos'd; thus

Say, As the propos'd Rate per Cent. is to 100*l.* so is the propos'd Yearly Rent, to his Principal, sought.

Then find (by Rule 8.) what Sum that Principal will be increas'd to, in the propos'd Number of Years, and at the propos'd Rate; and from that Sum subtract the Principal, and the Remainder is the Increase of the Annuity sought.

Example.

Suppose a Landlord live far from his Tenant, and yet judging his Tenant honest, and able to pay him; is content to forbear his Rent 14 Years, upon condition his Tenant will allow him 6*l.* per Cent. per Annum (Compound Interest) for the Forbearance; and suppose the Rent be 33*l.* per Annum. Now the Question is, What must the Landlord receive at the End of the 14 Years? Ans. 693*l.* 10*s.*

(11.) To find the present Worth of an Annuity or Yearly Rent, allowing Discount or Rebate, at any Rate per Cent. Compound Interest.

The RULE.

First find (by Rule 10.) the Increase or Amount of the propos'd Annuity, for the propos'd Time, and at the propos'd Rate.

Then find (by Rule 9.) the present Worth, of the Sum thus found due at the End of the propos'd Time; and that shall be the present Worth of the Annuity sought.

Ex-

Example.

What is a Lease of 25 *l.* per *Annum*, to continue for 21 Years, worth in present Money, Discount or Rebate (at 6 *l.* per Cent. Compound Interest) being allow'd to the Purchaser? *Ans.* 294 *l.* 1 *s.* 6 *d.* 1 *q.*

There are several other Cases in Interest, but these being the most useful ones; I shall (to avoid Volu-
mosity) wave the rest in this place.

C H A P. XIV.

Military Affairs or Stratarithmetry, by the Line.

P R O P. I.

To find the Number of Men contain'd in any Square or
or Oblong Battalia.

The R U L E.

AS 1, is to the Number of Men in Rank; so is the
Number of Men in File, to the Number of Men
in the Battalia requir'd. This needs no Example.

P R O P. II.

Any Number of Souldiers being propos'd; To Order them
into a Square Battalia; that is, there shall be as many
in Rank as in File.

The R U L E.

Extract the Square Root of the Given Number of
Souldiers, (by Chap. 11,) and that shall be the Num-
ber of Men in Rank and File requir'd.

Example.

Suppose it were requir'd to Order 1600 Men into a
Square Battalia; How many Men must there be in Rank
and File? *Ans.* 40 in each.

P R O P.

PROP. VI.

Any Number of Soldiers being propos'd; To Order them
in Double (or Triple, or Quadruple, or, &c.) Bat-
talia; that is, having twice (or three times; or four
times, or, &c.) as many in Rank as in File.

The R U L E.

Extract the Square Root of $\frac{1}{2}$ (or $\frac{1}{3}$, or $\frac{1}{4}$, or, &c.)
the given Number of men; and that is the Number
of Men in File; and Double (or Triple, or Quadruple,
or, &c.) that Number is the Number in Rank.

Example.

Suppose 1800 Men were to be placed in Battalia, so
as the Number of Men in Rank, might be double to
the Number in File; How many must there be in Rank,
and how many in File? Ans. 30 in File, and 60 in
Rank.

PROP. VII.

Any Number of Soldiers being propos'd to be put in Batta-
lia; with a certain Number assign'd to be put in { Rank; }
{ File; }

To find the Number in { File.
{ Rank.

The R U L E.

As the Number of Men assign'd to be put in { Rank, }
{ File, }
is to the whole Number of Soldiers; so is 1, to the
Number to be put in { Rank.
{ File.

Example.

Let it be requir'd to place 872 Men in Battalia, so
that there shall be 8 in File; How many must there
be in Rank? Ans. 109. For, As 8: 872:: 1: 109.

P R O P.

P R O P. V.

Any Number of Souldiers being propos'd; To place them in Battalia, so that the Number of Men in $\left\{ \begin{array}{l} \text{Rank,} \\ \text{File,} \end{array} \right\}$

shall be to the Number of Men in $\left\{ \begin{array}{l} \text{File,} \\ \text{Rank,} \end{array} \right\}$ according to any Proportion assign'd.

The R U L E.

As the Ratio respecting the $\left\{ \begin{array}{l} \text{Rank,} \\ \text{File,} \end{array} \right\}$ is to the Ratio respecting the $\left\{ \begin{array}{l} \text{File,} \\ \text{Rank,} \end{array} \right\}$ so is the whole Number of Souldiers, to a 4th. Whole Square Root is the Number of Men in $\left\{ \begin{array}{l} \text{File,} \\ \text{Rank,} \end{array} \right\}$

Having thus found the Number of Men to be placed in $\left\{ \begin{array}{l} \text{File,} \\ \text{Rank,} \end{array} \right\}$ Find the Number to be placed in $\left\{ \begin{array}{l} \text{Rank,} \\ \text{File,} \end{array} \right\}$ by Prop. IV.

Example.

Let it be requir'd to place 600 Men in Battalia, so that the Number of Men in Rank, may be to the Number in File, as 3 to 2. Now, how many must there be in Rank, and how many in File? Ans. 20 in File, and 30 in Rank.

P R O P. VI.

Any Number of Souldiers being given to be put in Battalia, and their Number and Distance in Rank and File, being given also; To find how much Ground they will occupy.

The R U L E.

As 1, is to the Distance in $\left\{ \begin{array}{l} \text{Rank,} \\ \text{File,} \end{array} \right\}$ so is the Number

F

ber

ber in $\left\{ \begin{array}{l} \text{Rank, less by 1,} \\ \text{File, less by 1,} \end{array} \right\}$ to a 4th. Then the Rectangle or Product of these 4ths, gives the quantity of Ground sought, in such Measures as the Distance in Rank and File were given in.

Example.

Let it be requir'd to place 600 Men in Battalia, so as there may be 30 in Rank, and 20 in File; and they all to stand at Order; (i. e. 2 Foot distant in Rank, and as much in File;) Now, how much Ground will they occupy? Ans. 4959 Square Feet: It being an Oblong piece of Ground, of 87 Feet long, and 57 Feet broad. For, As 1 : 3 :: so is $\left\{ \begin{array}{l} 29 \\ 19 \end{array} \right\}$ to $\left\{ \begin{array}{l} 87 \\ 57 \end{array} \right\}$ Then 87 multiply'd by 57, produceth 4959.

P R O P. VII.

The Hight of the Wall of a Fort or Castle being given, with the breadth of the Trench about the Wall; To find the Length of a Scaling-Ladder, that will reach from the Edge of the Trench to the Top of the Wall.

The R U L E.

Square the Hight of the Wall, and also the Breadth of the Trench; add these two Numbers together, and extract the Square Root of their Sum; and so shall you have the Length of the Scaling-Ladder requir'd.

Example.

Let the Hight of the Wall of a Fort be 16 Feet, and the Breadth of the Trench about it 12 Feet; How long must the Scaling-Ladder be? Ans. 20 Foot. For the Square of 16, is 256; and the Square of 12, is 144; their Sum is 400. Whole Square Root is 20.

CHAP. XV.

Duplicate Proportion, by the Line.

Duplicate Proportion is such a Proportion as is between Lines and Superficies, or between Superficies and Lines.

(1.) For the Proportion of *Lines* or *Superficies*; this is the

R U L E.

As the First Term, is to the Second of the same Denomination; so is the Third, at twice turning the Compasses, to the Fourth required. (Which Rule is thus to be understood. Extend the Compasses, from the first given Number, to the second of the same Denomination; that Extent being turn'd twice (either the same; or the contrary way, as Reason will direct,) from the third Number, will reach to the fourth required.

Example 1.

If the Diameter of a Circle be 14 Inches, and the Area thereof be 154 Inches; What is the Area of another Circle, whose Diameter is, 28 Inches? *Ans.* 616 Inches. For the Extent from 14 to 28, being turn'd twice (the same way) from 154, will reach to 616.

Example 2.

If a Piece of Land that is 20 Perches Square, be worth 80 *l.* How much is a Piece of Land (of the same goodness) worth, that is 30 Perches Square? *Ans.* 120 *l.* that is, 67 *l.* 10 *s.*

Example 3.

If a Fathom of Rope of 6 Inches Compass, weigh 6 lb. How much shall a Fathom of Rope of 12 Inches Compass weigh? *Ans.* 24 lb.

Example 4.

How many Acres of Land of our *English* Measure of 16.5 Foot to the Perch, are equal to 30 *Irish* Acres of 21 Foot to the Perch? Ans. 48.6 Acres.

Note. That in Duplicate Proportion, there are always three Terms or Num^{ers} given, to find a 4th; and of those three given Numbers, two are always of the same Denomination; and of those two, one is always a Term of Supposition, (which must be the first Term,) and the other of Demand, which must be the second Term; and the remaining given Term must be the third Term.

Then, if Reason shew that the $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ the second Term is, the $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ the 4th Term must be; then the Extent of the Compasses from the first to the second, must be turn'd the same way, from the third towards the fourth.

But, if Reason shew, that the $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ the second Term is, the $\left\{ \begin{smallmatrix} \text{lesser} \\ \text{greater} \end{smallmatrix} \right\}$ the fourth Term must be; then the Extent from the first to the second, must be turn'd the contrary way, from the third towards the fourth.

So that you see, some Questions are in Duplicate Proportion Direct, (as were the three first Examples;) and some in Duplicate Proportion Reverse, (as in the fourth Example.)

(2.) For the Proportion of Superficies to Lines; this is the

R U L E.

Extend the Compasses to half the Distance between the two given Numbers of the same Denomination; and that Extent (being apply'd the right way, according to the Note at the end of the first Rule of this Chapter;) shall reach from the other given Number, to the fourth required.

Ex-

Example 1.

Let there be two Circles given, the Area of one of them being 154 Inches, and its Diameter 14 Inches; and the Area of the other Circle 616 Inches; What is its Diameter? Ans. 28 Inches. For the Distance between 154, and 616, being divided into two equal Parts, that Extent of the Compasses will reach from 14 to 28.

Example 2.

There is a Piece of Land of 20 Perches square, worth 30 l. There is another Piece of Land (of the same Goodness) worth 67 l. 10 s. How many Perches square ought that Piece to be? Ans. 30.

Example 3.

If a Fathom of Rope of 6 Inches Compass, weigh 6 lb. How much will the Compass of that Rope be, of which a Fathom weighs 24 lb.? Ans. 12 Inches.

Example 4.

If a Piece of Land contain 30 Acres, being measur'd by the Irish Pole of 21 Foot; How long must the Pole or Perch be, by which the same Piece of Land being measur'd, it will contain 48.6 Acres? Ans. It must be 16.5 Foot: which is the English Pole or Perch.

(3.) There is another way of working Duplicate Proportion; which tho it be more tedious by the Line, yet, because it expresses something of the Nature of the Rule, I shall here exhibit it.

The R U L E.

Having found (by the Note at the End of the first Rule) which is the first and second Term: Then, for the Proportion of { Lines to Superficies } say, As the Square of the first, is to the Square of the second; So the Square Root of the first, is to the Sq. Root of the 2d; So is the third, to the fourth, sought.

(4.) The Effects of Duplicate Proportion are many; I shall here only hint at some of them.

The first and general Effect of Duplicate Proportion is; By having three Numbers given; To find a fourth in Duplicate Proportion: that is, that the third Number given; may be to the fourth sought; Either, (1.) As the Square of the first, is to the Square of the second (of the same Denomination). Or, (2.) As the Square Root of the first, is to the Square Root of the second.

The first of these is properly call'd *Duplicate Proportion*; and the Rule for it, is laid down in Rule 1. of this Chapter. But the second, tho' it (be comprehended under, and) be also commonly call'd *Duplicate Proportion*; yet is it more properly call'd *Subduplicate Proportion*. The Rule to work it, is laid down in the second Rule of this Chapter.

2. The second Effect is; By having the Diameter of a Circle (or the Side of any regular Polygon) given, with its Area; and the Diameter of another Circle (or the Side of another regular Polygon of the same kind) also given, to find its Area. This was Exemplify'd in Rule 1. Example 2.

The third Effect is, By having the Diameter of any one Circle (or the Side of any one regular Polygon) given, with its Value or Worth; and the Diameter of another Circle (or the Side of another regular Polygon, of the same kind) also given, to find its Value or Worth. This was Exemplify'd in Rule 1. Example 2.

4. The fourth Effect is, By having the Diameter or Circumference of any one Cylinder (or the Side or Circumference of any one regular Polygonal Prism) given, with its Gravity or Weight; and the Diameter or Circumference of another Cylinder (or the Side or Circumference of another Prism of the same kind) of the same length, also given; To find its Gravity or Weight. This was Exemplify'd in Rule 1. Example 3.

(4.)

5. The

5. The fifth Effect is, By having the Weight and Value of one Diamond (or other Precious Stone) given; and the Weight of another Diamond (or other Precious Stone) being also given; To find its Value. This has not been Exemplify'd; but it may be wrought either by Rule 1, or according to Rule 3, by this

ANALOGY.

As the Square of the Weight of one Diamond (or other Precious Stone) is to the Square of the Weight of another; so is the Value of the first, to the Value of the last.

Note. A good Diamond weighing 1 Carat, (i.e. 4 Grains,) is valu'd at 150 French Livres. But if the Diamond be not perfect, then they allow but 80 Livres to a Carat. Therefore,

(5.) To find the Value of any Diamond, this is the

RULE.

As 1, is to the Square of the Weight (in Carats) of any Diamond; so is $\left\{ \begin{smallmatrix} 150 \\ 80 \end{smallmatrix} \right\}$ French Livres, (for

$\left\{ \begin{smallmatrix} \text{a good one,} \\ \text{an imperfect one,} \end{smallmatrix} \right\}$ to the Value (in French Livres) of the given Diamond.

Note. A French Liver is about 1 s. 6 d. English.

There are many more Effects of Duplicate Proportion, which for Brevity sake I omit. The Reader may easily deduce 4 others from the Examples of the second Rule, the 3^d first being just contrary to the second, third, and fourth Effect.

C H A P. XVI.

Triplicate Proportion, by the Line.

Triplicate Proportion is such a Proportion as is between *Lines* and *Solids*, or between *Solids* and *Lines*. Thus 'tis commonly taken; tho more properly, the former would be call'd Triplicate, and the later Subtriplicate Proportion.

(1.) For the Proportion of *Lines* to *Solids*, this is

The R U L E.

As the first given Number, is to the second of the same Denomination; so is the third (at three times turning the Compasses), to the fourth requir'd.

Therefore,

Always extend the Compasses from the first given Number, to the second, of the same Denomination; that Extent of the Compasses, being turn'd three times the right way, (according to the Nature of the Question, which Reason will direct,) from the third Number, will reach to the fourth requir'd.

Example 1.

If a Bullet, whose Diameter is 4 Inches, weigh 9 lb. What will another Bullet of the same Metal weigh, whose Diameter is 8 Inches? Ans. 72 lb. For the Extent of the Compasses from 4, (the Diameter of the Bullet whose Weight is given,) to 8, (the Diameter of the Bullet whose Weight is sought; will reach (being three times turn'd the same way) from 9, (the Weight of the Bullet known,) to 72, the Weight of the Bullet sought.

Example 2.

If the Diameter of a Globe be 1 Inch, and the Solidity thereof be .5238 of an Inch; What is the Solidity of another Globe, whose Diameter is 10 Inches? Ans. 523.8 Inches; that is, almost 524 Inches.

Exam-

Example 3.

If a Globe of Silver of 1 Inch Diameter, be worth 144 s. (for so it will be at 5 s. the Ounce;) What will another Globe of Silver be worth, whose Diameter is 4 Inches? Ans. 921,6 s. that is, 46 l. 1 s. 7½ d.

Example 4.

If a Saker of 3.75 Inches Diameter in the Bore, require 4 lb. of Powder for its due Charge; What will a Demi-canon of 6.5 Inches Diameter in the Bore, require for its Charge? Ans. 20.8 lb. *fré.*

Example 5.

If a Brass Saker, whose Diameter is 11.5 Inches, do weigh 1000 lb. What will another Piece of Ordnance (of the same Metal and Shape) weigh, whose Diameter is 8.75 Inches? Ans. 837 lb.

Example 6.

If a Ship, whose Burden is 300 Tun, be 75 Feet in the Keel; What shall be the Burden of another Ship (of the same Mold) whose Keel is 100 Feet? Ans. 713 Tun.

(2.) For the Proportion of Solids to Lines, this is

The RULE.

Extend the Compasses to one third of the Distance between the two given Numbers of the same Denomination; and that Extent of the Compasses being apply'd the right way, (which Reason will direct,) will reach from the other given Number, to the fourth requir'd.

Example 1.

If a Bullet, whose Diameter is 4 Inches, weigh 9 lb. What is the Diameter of another Bullet, (of the same Metal) whose Weight is 72 lb.? Ans. 8 Inches. For the Distance between 9 and 72, being divided into three equal Parts; that Extent of the Compasses, will reach from 9 to 81.

(A)

Example

Example 2.

If the Diameter of a Globe be 1 Inch, and the Solidity thereof be .5238 of an Inch; What is the Diameter of another Globe, whose Solidity is 523.8 Inches?

Ans. 10 Inches.

Example 3.

If a Globe of Silver, of 1 Inch Diameter, be worth 14.4s. What will be the Diameter of another Globe of Silver, that is worth 921.6s.?

Ans. 4 Inches.

Example 4.

If a Saker of 3.75 Inches Diameter in the Bore, require 4 lb. of Powder for its due Charge; What will be the Diameter (in the Bore) of a Demi-Canon, that requires 20.8 lb. of Powder for its Charge?

Ans. 6.5 Inches.

Example 5.

If a Brass Saker, whose Diameter is 11.5 Inches, do weigh 1800 lb. What will be the Diameter of another Piece of Ordnance, (of the same Metal and Shape) whose Weight is 837 lb.?

Ans. 8.75 Inches.

Example 6.

If a Ship, whose Burden is 300 Tun, be 75 Feet in the Keel; What shall be the Length of the Keel of another Ship, (of the same Mold) whose Burden is 713 Tun?

Ans. 100 Feet.

(3.) There is another way of working Triplicate Proportion; which, tho it be more tedious by the Line, yet, because it gives some Light to the Nature of the Rule, I shall here exhibit it.

The R U L E.

First find (by the Note in Rule 1, of the preceding Chapter,) which is the first and second Term. Then, for the Proportion of $\left\{ \begin{array}{l} \text{Lines to Solids,} \\ \text{Solids to Lines,} \end{array} \right\}$ say, As the $\left\{ \begin{array}{l} \text{Cube of the first, to the Cube of the second;} \\ \text{Cube-Root of the 1st, to the Cube-Root of the 2d;} \end{array} \right\}$ so is the 3d, to the 4th sought.

(4.) The

(4.) The Effects of Triplicate Proportion, are

1. The first and general Effect is, By having three Numbers given; To find a fourth in a Triplicate Proportion; that is, that the third Number given, may be to the fourth sought; Either, (1.) As the Cube of the first, is to the Cube of the second; Or (2.) As the Cube-Root of the first, is to the Cube-Root of the second.

The Rule for the $\left\{ \begin{smallmatrix} \text{first} \\ \text{second} \end{smallmatrix} \right\}$ is laid down in the $\left\{ \begin{smallmatrix} \text{first} \\ \text{second} \end{smallmatrix} \right\}$ Rule of this Chapter. Or, either of them may be wrought by the third Rule.

2. } The $\left\{ \begin{smallmatrix} 2d \\ 3d \\ 4th \end{smallmatrix} \right\}$ Effect is, By having the Diameter

3. } of any one Globe (or, the Side of any one of the Platonick Bodies) given, with its $\left\{ \begin{smallmatrix} \text{Gravity or Weight;} \\ \text{Solidity;} \\ \text{Value or Worth;} \end{smallmatrix} \right\}$

4. } and the Diameter of another Globe (or the Side of another Platonick Body, of the same kind) also given; To

find its $\left\{ \begin{smallmatrix} \text{Gravity.} \\ \text{Solidity.} \\ \text{Value.} \end{smallmatrix} \right\}$ This was Exemplify'd in Example

the $\left\{ \begin{smallmatrix} \text{first} \\ \text{second} \\ \text{third} \end{smallmatrix} \right\}$ of the first Rule of this Chapter.

5. The fifth Effect is, By knowing the Quantity (or Weight) of Powder, that will load one Piece of Ordnance, the Diameter of whose Bore is given; To find the Quantity (or Weight) of (the same sort of) Powder, that will load another Piece of Ordnance, the Diameter of whose Bore is also given. This was Exemplify'd in Rule 1. *Examp. 4.*

6 The sixth Effect is, By knowing the Diameter and Weight of any one Piece of Ordnance; To find the Weight of another Piece of Ordnance, (of the same

same Metal and Shape) whose Diameter is also given. This was Exemplify'd in Rule 1. Example 5.

7. The seventh Effect is, By knowing the Length of the Keel, Breadth at the Beam, or Depth in the Hold, of any one Ship, with its Burthen; To find the Burthen of another Ship, (of the same Mold) whose Length in the Keel, Breadth in the Beam, or Depth in the Hold, is also given. This was Exemplify'd in Rule 1. Example 6.

Many are the Effects of Triplicate Proportion; but for Brevity I omit the rest. The Reader may easily (*per se*) deduce six other Effects, from the six Examples of the second Rule of this Chapter; they being exactly contrary to the second, third, fourth, fifth, sixth, and seventh Effects already mention'd.

So much shall suffice (at this time) for the Use of the Line of Numbers in Arithmetick. I shall now proceed (*Deo juvante*) to treat in

BOOK II.

Of the Use of the Line of Numbers in Geometry.

PROEME.

1. I Am not here to teach the whole Art of Geometry; (for that would take up a far larger Volume alone, than this is design'd to be;) but only to shew how the most usual, and most useful Propositions, may be perform'd by the Line of Numbers.

2. I shall therefore suppose the Reader to be well seen in all the Definitions of Lines, Superficies, and Solids.

3. Having already (in Book 1. Chap. 6.) laid down Rules sufficient for the Resolution of any Analogy; I shall, in all the following Pages of this Book, lay down the Rules Analogically; or in such Tables as are easily reducible to Analogies.

4. I shall here consider Geometry, as it may (properly) be distinguish'd into these four General Heads, or Parts; viz.

{ 1 2 3 4 }	The Mensuration of	Line.
		Superficies.
		Solids.
		Concaves.

5. Under the first Head, I shall consider, (1.) The Proportion between the Diameters (or Circumferences) of Circles, and the Sides of Regular Polygons; and that either in respect of Equality, or of their being inscribed in,

in, or circumscribed about one another. (2.) The Proportion between the Diameters of Spheres, and the Sides of the Platonic Bodies; and that either in respect to their Equality, or their Inscription in, or Circumscription about one another. And (3.) The taking of Hights and Distances.

6. Under the second Head, I shall consider, (1.) The Mensuration of Superficial Figures. (2.) The Mensuration of the Superficiality of Solid Bodies. And (3.) The Application of Superficial Measure to the Mensuration of Artificer's Work.

7. Under the third Head, I shall consider, (1.) The Mensuration of Solid Bodies; Whether they are Irregular, simply Regular, or the Platonic Bodies. (2.) The Application of Solid Measure to the Mensuration of Artificer's Work. And (3.) Statics; or the Art of finding the Gravity of any Body. It is here done only by its Dimensions: A thing (to my knowledge) never treated of by any.

8. Under the fourth Head I shall consider the Art of Gauging.

CHAP. I.

The Mensuration of Lines, by the Line of Numbers.

PROP. I.

To find the Circumference of a Circle, the Diameter being given.

The RULE.

As $\left\{ \begin{array}{l} 7 \text{ to } 22; \\ 1 \text{ to } 3.14159; \\ 113 \text{ to } 355; \end{array} \right\}$ so is the Diameter } *Archimedes.*
 } the Circumference, } *Van Culen.*
 } according to } *Metius.*

Which last Analogy is accounted the most exact; tho there be little difference betwixt any of them.

PROP.

PROP. II.

To find the Circumference of a Circle, the Area being given.

The RULE.

As 7 to 88; so is the Area, to the Square of the Circumference. Therefore, the Analogy being wrought, Extract the square Root of the 4th Term, (by Book I. Chap. XI.) and that Root is the thing sought.

PROP. III.

To find the Diameter of a Circle, the Circumference being given.

The RULE.

As $\left\{ \begin{array}{l} 22 \text{ to } 7; \\ 3.14159 \text{ to } 1; \\ 355 \text{ to } 113; \end{array} \right\}$ so is the Circumference, to the Diameter, according to Archimedes, Van Culen, Metius.

PROP. IV.

To find the Diameter of a Circle; the Area being given.

The RULE.

As 11 to 14; so is the Area, to the Square of the Diameter. Therefore, the Analogy being wrought, Extract the Square Root of the 4th Term, (by Book I. Chap. XI.) and that Root is the thing sought.

These four Propositions are so very plain and easy, that they need no Examples.

PROP. V.

The Diameter of a Circle being given; To find the Side of any regular Polygon (from the Trigon to the Decagon) that may be either inscribed in, or circumscribed about, or is equal to, the given Circle,

Here are 24 Propositions, which (for Brevity sake) I have compriz'd in one; because a particular Exposition of them, would too much increase the intended Bulk of the Book. I have therefore here exhibited a brief Table, from whence the 24 Analogies may easily be deduc'd.

The TABLE.

The Diameter of a Circle being 1, the Side of a	That may be inscrib'd in that Circle, is	That may be circumscrib'd about that Circle, is	That is e- qual to the Circle, is
Trigon	0.86603	1.73205	1.34704
Tetragon	0.70711	1.00000	0.88623
Pentagon	0.58778	0.72654	0.67578
Hexagon	0.50000	0.57735	0.54992
Heptagon	0.43390	0.48160	0.46500
Octagon	0.38268	0.41421	0.40393
Nonagon	0.34202	0.36397	0.35651
Decagon	0.30901	0.32491	0.31955

Therefore, to find any of the *Quæsitæ* (or Things sought) in this Proposition, this is

The RULE.

As 1, is to the respective Number in the Table; so is the Diameter of any Circle given, to the *Quæsitum* or Thing sought.

Example {
1.
2.
3.

What is the Side of a Trigon (or Equilater-Triangle) that { may be inscribed in
may be circumscribed about } a Circle, whose
is equal to

Diameter is 7? Ans. The Side of the { Inscribed
Circumscribed }
Equal

Trigon, is { 6. 06221.
12. 12435.
9. 42928.

Example {
4.
5.
6.

What

What is the Side of a Tetragon (or Geometrical Square) that $\left\{ \begin{array}{l} \text{may be inscribed in} \\ \text{may be circumscribed about} \\ \text{is equal to} \end{array} \right\}$ a Circle, whose Diameter is 7? Ans. The Side of the $\left\{ \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ Tetragon, is $\left\{ \begin{array}{l} 4.94977. \\ 7.00000. \\ 6.26361. \end{array} \right.$

And thus, as these six Propositions have been wrought, in the six preceding Examples; so, after the same manner, are all the other 18 to be wrought.

Note 1. Tho' all the Analogies in the preceding Table are exact enough for Common Practice; yet, it must be confess'd, that some of them are not absolutely, exact; neither indeed, is it possible they should be so; the Reason is, Because the *Datum* (or Thing given) being a Rational Number, many of the *Quæsitæ* (or Things sought) will be Irrationals; that is, Numbers (if I may so call them) that cannot be express'd by any Numbers, Integral, or Fractional, but only by their Squares, or other higher Powers. E. g. If the Diameter of a Circle be Rational; the Side of its Inscribed, or Circumscribed Trigon, will be Irrational; but their Squares may be express'd in Numbers. And so of others.

Note 2. If the Reader be ignorant of Decimals, (which yet he ought not to be) he may reduce the (many) Analogies that are (in this Book) Decimally express'd, into the Common Account, by annexing so many Cyphers to the Unity, as there are Figures to the Right Hand of the Point in the Decimal Number. E. g. (In the preceding Table) Instead of saying, As 1, to 1.34704, he may say, As 10, to 13.4704; or, As 100, to 134.704; or, As 1000, to 1347.04; or, As 10000, to 13470.4; or, lastly, As 100000, to 134704. But 3 or 4 Places being nice enough for Common Practice, (especially by the Line) the Reader need take no more; rejecting all the rest as useless; only if the next Decimal Figure

exceed 5, he may add 1 to the last of the Figures he takes in. Thus, instead of saying, As 1, to 1.34704, he may say, As 100, to (not 134, but) 135; because the next Decimal Figure is 7.

If the two Ratio's be both Decimal Numbers, then remove the Points further to the Right Hand equally, in both Ratio's. Thus, if the Analogy were, As 2.34704, to .88623, you may say, (for 'twill be the same thing) As 134704, to 88623. Or (making the Numbers less, by the former Part of this Note,) as 135, to 89.

P R O P. VI.

The Circumference of a Circle being given; to find any of the Quæsitæ in the preceding Proposition.

Here also (to avoid a Tedium) are 24 Propositions exhibited in one; and all the 24 Analogies may be deduced from the Table in Prop. 5. by the following

R U L E.

First find the Diameters of the given Circle, by Prop. 3. and then find any of the Quæsitæ, by Prop. 5.

Or more briefly, thus,

Say, as 3.14159, is to the Number (in the Table) respecting the Quæsitum; so is the Circumference of the given Circle, to the Quæsitum.

Thus, If the Circumference of the given Circle were

21.99115, the Sides of its $\left. \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ Trigon,

and Tetragon, will be found to be the same as in the Examples of the preceding Proposition. The Reason is plain, because this Circle (according to *Metius's* Analogy) has the same with that.

This being so very plain and easy, I think it needs no farther Exemplification.

P R O P.

P R O P. VII.

The Side of any regular Polygon (from the Trigon to the Decagon) being given; To find the Diameter, or Circumference of its $\left\{ \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ *Circle.*

Here (to avoid Tedium) I have compriz'd 48 Propositions in one; all whose several Analogies are also deducible from the Table in Prop. 5. by the following RULES.

1. For the Diameter,

Say, As the respective Number (in the Table) under the Title $\left\{ \begin{array}{l} \text{Inscribed,} \\ \text{Circumscribed,} \\ \text{Equal,} \end{array} \right\}$ is to 1; so is the given Side of the Polygon, to the Diameter of its $\left\{ \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ Circle.

2. For the Circumference,

Say, As the respective Number (in the Table) under the Title $\left\{ \begin{array}{l} \text{Inscribed,} \\ \text{Circumscribed,} \\ \text{Equal,} \end{array} \right\}$ is to 3.14159; so is the given Side of the Polygon, to the Circumference of its $\left\{ \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ Circle.

Example.

If the Side of a Trigon be 1; What is the Diameter and Circumference of his Inscribed, Circumscribed, and Equal Circle? Ans. The Diameter of its $\left\{ \begin{array}{l} \text{Inscribed} \\ \text{Circumscribed} \\ \text{Equal} \end{array} \right\}$ Circle, is $\left\{ \begin{array}{l} .577, \text{ \& c.} \\ 1.154, \text{ \& c.} \\ .742, \text{ \& c.} \end{array} \right.$

And

And the Circumference of its { Incribed,
Circumscribed } Circle,
Equal

is { 1. 81, &c.
3. 62, &c.
2. 33, &c.

P. R. O P. VIII.

The Side of any regular Polygon (from the Trigon to the Decagon) being given; To find the Side of any other regular Polygon (from the Trigon to the Decagon) equal thereto.

Here (to avoid Prolixity) are 56 Propositions, which I have included in one; viz. 7 relating to each Polygon. e.g. To find a Tetragon, a Pentagon, a Hexagon, &c. equal to a given Trigon. So also, To find a Trigon, a Pentagon, a Hexagon, &c. equal to a given Tetragon. And so of all the other Polygons in the Table in Prop. 5. from which Table all these 56 Propositions are also deducible, by the following

R U L E.

Say, As the Number (under the Title *Equal*) against the given Polygon, is to the Number (under the same Title) against the Polygon, whose Side is sought; so is the Side of the given Polygon, to the Side of the Polygon sought.

Example 1.

Let it be requir'd to find a Tetragon (or Geometrical Square) equal to a Trigon (or Equilateral Triangle) whose Side is 10. Here, I say, (from the Table) As 1.34704, is to .88623; so is 10 (the Side of the given Trigon), to 6.5790, the Side of the Tetragon sought.

Example 2.

Let it be requir'd to find a Trigon, equal to a Tetragon, whose Side is 6.5790. Here, I say, As .88623, is to 1.34704; so is 6.5790, to 10, the Side of the Trigon sought.

These

These two Examples (I think) are sufficient to make the Rule intelligible to any ordinary Understanding.

P R O P. IX.

To find the Perpendicular of a Trigon (or Equaliter Triangle) the Side only being given.

The R U L E.

As 1, is to .86603; so is the Side of a Trigon, to its Perpendicular.

Example.

If the Side of a Trigon be 10; What is the Length of its Perpendicular? Ans. 8.6603.

P R O P. X.

To find the Diagonal of a Tetragon (or Geometrical Square.)

The R U L E.

As 1, is to 1.41774; so is the Side of a Tetragon, to its Diagonal.

Example.

If the Side of a Tetragon be 10; What is the Length of its Diagonal? Ans. 14.1774.

P R O P. XI.

Having the Chord and Versed-Sine of an Arch given; To find the whole Diameter of that Circle, of which the given Arch was a part.

The R U L E.

As the Versed-Sine, is to 1; so is the Square of half the Chord, to a 4th. Which 4th added to the Versed-Sine, gives the whole Diameter of the Circle.

Example.

Let there be a part of a Cheese, (cut off by a Right Line) whose Chord (i. e. the Right Line by which it was cut

cut off) is 12 Inches, and its *Versed-Sine* (i. e. the Perpendicular from the middle of the Straight Side, to the middle of the Arch) is 4 Inches; What was the whole Diameter of that Cheese? *Ans.* 13 Inches.

P R O P. XII.

To find the Parts of a Line, or Number, divided by Extream and Mean Proportion.

The R U L E.

As 1, is to the given Number; so is $\left\{ \begin{smallmatrix} .618 \\ .382 \end{smallmatrix} \right\}$ to the $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ Part of the Line or Number.

Example.

What are the Parts of 1000, divided by Extream and Mean Proportion? *Ans.* The $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ Part is $\left\{ \begin{smallmatrix} 618. \\ 382. \end{smallmatrix} \right\}$

Note. A Line, or Number, is said to be divided by Extream and Mean Proportion; when the whole Line or Number is (in proportion) to the greater Part, as the greater Part is to the lesser.

P R O P. XIII.

To find any Chord in a Circle, that shall cross the Diameter (at Right Angles) in any Point assigned.

The R U L E.

If the given Part of the Diameter (by which the Chord is to pass) be $\left\{ \begin{smallmatrix} \text{greater} \\ \text{lesser} \end{smallmatrix} \right\}$ than the Radius, (or Semidiameter of the Circle,) then subtract $\left\{ \begin{smallmatrix} \text{the Radius from it,} \\ \text{it from the Radius,} \end{smallmatrix} \right\}$ and the Remainder is the Co-Sine of half the Arch cut off by the Chord.

Then,
From the Square of the Radius, subtract the Square
of

of the Co-Sine; and the Remainder is the Square of half the Chord sought.

Example.

If the whole Diameter of a Circle be 13 Inches; What is the Length of the Chord, crossing the Diameter (at Right Angles) at the Distance of 4 Inches, from one End of the Diameter? Ans: 12 Inches.

P R O P. XIV.

To find the Trigonal, Tetragonal, Pentagonal, Hexagonal, Heptagonal, Octagonal, Nonagonal, and Decagonal Chords, of any Circle whose Diameter is given.

Here are 8 Propositions, which may all be solved by Prop. 5. foregoing; The Trigonal, Tetragonal, &c. Chords of a Circle, being the same with the Side of its inscribed Trigon, Tetragon, &c.

P R O P. XV.

The Diameter of a Sphere being given: To find the side of any of the Platonic Bodies that may be either
 { Inscribed in
 Circumscribed about } the given Sphere.
 or is Equal to

Here are 15 Propositions which (for Brevity sake) I have Compriz'd in one; all whose several Analogies, (with many more used in the following Propositions) may be deduced from this following

T A B L E.

The Diameter of a Sphere being Unity, the Side of a	That may be Inscribed in the Sphere, is	That may be Circumscribed about the Sph. is,	That is Equal to the Sph. is
Tetraëdron.	0.816497	2.44948	1.64417
Octaëdron.	0.707107	1.22474	1.03576
Hexaëdron.	0.577350	1. —	0.80610
Icosaëdron.	0.525731	0.66178	0.62143
Dodecaëdron.	0.356822	0.44903	0.40883

Now,

Now,

To find any of the *Quæsitæ* (or Things sought) in this Proposition; this is

The R U L E.

Say, As 1, is to the Number (in the Table) respecting the *Quæsitum* (or Thing sought); so is the Diameter of the given Sphere, to the *Quæsitum*.

Example 1.

What is the Side of a Tetraedron, that may be inscribed in a Sphere, whose Diameter is 10? Ans. 8.16497. For, As 1, (the supposed Diameter of a Sphere) is to .816497; (the Side of a Tetraedron that may be inscribed in that Sphere, whose Diameter is 1,) so is 10 (the Diameter of the given Sphere) to 8.16497 (the Side of the Tetraedron that may be inscribed in that Sphere whose Diameter is 10.)

Example 2.

What is the Side of a Tetraedron, that may be circumscribed about a Sphere, whose Diameter is 10? Ans. 24.4948. For, As 1: 2.44948 :: 10: 24.4948.

Example 3.

What is the Side of a Tetraedron, that is equal (in Solidity) to a Sphere whose Diameter is 10? Ans. 16.4417. For, As 1: 1.64417 :: 10: 16.4417.

Thus I have Exemplify'd 3 of the 15 Propositions; the other 12 are to be wrought after the same manner.

Note. If the Reader be so dull, as not to apprehend the Analogy already laid down, (for finding the *Quæsitæ* in this Proposition,) let him take it in other words, thus;

As 1, is to the Number (in the Table) against any of the Platonic Bodies, and under the Title $\left\{ \begin{array}{l} \text{Inscribed;} \\ \text{Circumscribed;} \\ \text{Equal;} \end{array} \right\}$ so

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so is the given Diameter of any Sphere, to the Side of that respective Platonic Body when it is
 { inscribed in
 { circumscribed about } the given Sphere.
 { equal to

PROP. XVI.

The Circumference of a Sphere being given: To find any of the Quæsitæ in the 15th Proposition.

Here are also 15 Propositions, which may all be solved by (the help of the Table in Prop. 15. and) the following

RULE.

First find the Diameter of the given Sphere by Prop. 3 of this Chapter 3 and then find any of the Quæsitæ by Prop. 15.

Or more briefly thus,

Say, As 3.14159, is to the Number (in the Table) against any of the Platonic Bodies, and under the

Title { Inscribed;
 { Circumscribed } so is the given Circumference
 { Equal;

of any Sphere, to the Side of that respective Platonic Body when it is { inscribed in
 { circumscribed about } the given
 { equal to
 Sphere.

Example { 1.
 { 2.
 { 3.

If the Circumference of the Sphere be 31.4159;
 What is the side of its { Inscribed
 { Circumscribed } Tetraedron?
 { Equal

Answ. { 8.16497 }
 { 24. 4948 } the same that was found in the
 { 16. 4417

H

Exam-

Examples of the preceding Proposition. The Reason is, because this Sphere has the same Diameter with that.

P R O P. XVII.

The Side of any one of the 5 Platonic Bodies being given; to find the Diameter, or Circumference of its
Inscribed
Circumscribed } *Sphere.*
Equal

To avoid Prolixity, I have included 30 Propositions in one; which may be all solved by help of the Table in Prop. 15. and the following Rules.

1. If the Diameter of the Sphere be sought, this is

The R U L E.

As the respective Number (in the Table) under the
Inscribed,
 Title { *Circumscribed,* } is to 1; so is the given Side of
Equal,

the Platonic Body, to the Diam. of its { *Inscribed*
Circumscrib. }
Equal
 Sphere.

2. If the Circumference of the Sphere be sought, this is

The R U L E.

As the respective Number (in the Table) under the
Inscribed,
 Title { *Circumscribed,* } is to 3.14159; so is the gi-
Equal,
 ven Side of the Platonic Body, to the Circumference
Inscribed
 of its { *Circumscribed* } Sphere.
Equal

Example { 1.
 2.
 3.

if

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If the Side of a Tetraedron be 105, What is the Diameter and Circumference of its {
 Incribed
 Circumscribed
 Equal
 Sphere? Ans. The Diameter of its {
 Incribed
 Circumscribed
 Equal

Sphere is { 4.084 } and the Circumference of its
 12.24
 6.08

{ Incribed
 Circumscribed
 Equal } Sphere is { 12.82
 38.47
 19.10

P R O P. XVIII.

The Side of any one of the Platonic Bodies being given
 To find the Side of any of the other Platonic Bodies
 equal (in Solidity) thereto.

Here are 20 Propositions compriz'd in one; viz.
 4 relating to each Platonic Body: e.g. To find any
 Octa-, Hexa-, Icosa-, and Dodecaedron, equal to a
 given Tetraedron. So likewise To find a Tetra-, Hexa-,
 Icosa- and Dodecaedron, equal to a given Octaedron.
 And so of the rest. And all these 20 Propositions are solv-
 able from the Table in Prop. 15. by the following

R U L E.
 As the Number (under the Title *Equal*) against the
 given Platonic Body, is to the Number (under the same
 Title) against the Platonic Body whose Side is sought;
 so is the Side of the Platonic Body given, to the Side
 of the Platonic Body sought.

Example.

Let it be requir'd to find the Side of a Tetraedron,
 that shall be equal (in Solidity) to a Dodecaedron, whose
 Side is 10.

Here I say, (from the Table) As 140883, is to 1.64417;
 so is 10 the given Side of the Tetraedron, to 40.21
 the Side of the Tetraedron sought.

H 2

Thus,

That, kind Reader, I have (in this Chapter) given thee Rules for the Solution of 249 Propositions; (viz. 160 from the Table in Prop. 57 & 80 from the Table in Prop. 15, and 9 others;) and all, I think, as plain and intelligible, (even to the meanest Capacity, that is but diligent and industrious;) as if I had said ten times as much.

I should now say something of taking Heights and Distances; but, I think, this Chapter is already long enough; I shall therefore refer it to the next.

CHAP. II.

Of Altimetry, and Longimetry: or the Usefulness of the Line of Numbers, in taking Heights and Distances.

I shall not here teach the whole Art of taking Heights and Distances; for in so doing I shall step beyond the Bridge: I shall therefore keep to my Title, by showing only the Usefulness of the Line of Numbers in this Affair.

Heights and Distances may be taken by different ways, almost innumerable, both with and without Instruments. Of Instruments I shall mention only the Quadrant; because that (as it may be, and is commonly made,) is the most simple Instrument in the Mathematics, and well known almost to every one.

SECT. I.

Of taking Heights by the Quadrant.

Heights are taken by the Quadrant, either by the Degrees on the Limb, or by the Divisions on the Quadrant. Where Note,

1. That if you work by the Degrees on the Limb, you

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you must make use of the Table of Natural Sines and Tangents;

2. That if you work by the Quadrant; then this Scale (of the Quadrant) which is next to the Sights (of the Quadrant, is (by me) call'd the Scale of Right Shadows; and the other I call the Scale of Contrary Shadows. This I thought fit to specify, because Authors differ in their Denomination. So this being premis'd, I proceed to

P R O P. I.

To take the Height of an accessible Object, by the Quadrant of the Quadrant.

The R U L E.

(1.) If (when looking through the Sights of the Quadrant, at the Apex of the Object) the Thread fall on the Right Shadow; say, As the Parts cut by the Thread, is to the Radius of the Quadrant, (i.e. the whole Number each Side of the Quadrant is divided into); so is the Distance from the Station to the Object, to the Height of the Object above the Horizontal Line of the Eye. But, (2.) If the Thread fall on the contrary Shadow, then say, As the Radius of the Quadrant, is to the Parts cut by the Thread; so is the Distance from the Object to the Station, to the Height of the Object above the Eye, as before.

Example { 1.

Suppose the Radius of the Quadrant be 100, and the Parts of the { Right } Shadow cut by the Thread 50; and the Distance of the Object from the Station { 40 } Foot; What is the Height of the Object? { 160 }

Ans. 80 Foot above the Level of the Eye. For,

$$\text{As } \begin{cases} 50 : 100 :: 40 : 80. \\ 100 : 50 :: 160 : 80. \end{cases}$$

11th. The Distance from the Station to the Object, must be taken, to a Perpendicular under that Point of the Object, seen through the Sight, when the Observation was made; which in Sceptles, &c. will be the middle of the Object; but in such things as are flat on the Top, it will be that side next the Station.

2. That having made Observation at the Apex of the Object; you should also make an Horizontal Observation; making a Mark on the Object where the Visual Ray falls, and measuring the Distance from this Mark to the Ground; add it to the Height before found, so shall you have the true Height of the Object from the Ground.

R R Q P. II.

Another way to take the Height of an accessible Object, by
the Quadrant of the Quadrant.

THE RULE:

If (the Sun shining through the Sights of the Quadrant) the Thread fall on the { Right } Shadow;
 { Contrary }
 As the { Parts cut, is to the Radius of the Quadrant; }
 So, As the { Radius of the Quadrant, is to the Parts cut; }
 So is the Length of the Shadow of the Object, to the
 Height of the Object above the Ground.

Example $\begin{cases} 1. \\ 2. \end{cases}$

Let the Radius of the Quadrant be 100, and the Parts of { Right } Shadow cut by the Thread, 50, { Contrary } and the Length of the Shadow { 30 } Feet; What is the Height of the Object? Ans. 60 Feet. For, As { 50 : 100 :: 30 : } 60, { 100 : 50 :: 120 : }

Note. For taking Heights this way, such a Time of the Day should be chosen, as that the Shadow of the Object

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Object may fall (as near as may be) upon the level Ground: For if the Ground whereon the Shadow falls, do (considerably) recline from, or incline towards the Object; as the Shadow will be thereby longer or shorter, so will the Height of the Object be found more or less than the Truth.

P. R. O P. III.

To take the Height of an inaccessible Object, by the Quadrant of the Quadrant.

The R U L E.

This Proposition is perform'd by two Stations; at each of which, take a View of the Apex of the Object, through the Sights of the Quadrant: then, if (in any of the Stations) the Thread fall upon the $\left. \begin{matrix} \text{Right} \\ \text{Contrary} \end{matrix} \right\}$ Shadow; say, As the $\left. \begin{matrix} \text{Radius of the Quadrant, is to the} \\ \text{Parts cut, is to the Radius of the} \end{matrix} \right\}$ Parts cut; } so is x , to a 4th. Having thus found a 4th Proportional for each Station; say, As the Difference of these 4ths, is to x ; so is the Distance betwixt the two Stations, to the Height of the Object.

Example $\left. \begin{matrix} 1. \\ 2. \end{matrix} \right\}$

Suppose the Radius of the Quadrant be 100; and that at the first Station, the Parts cut by the Thread, be $\left. \begin{matrix} 50 \\ 20 \end{matrix} \right\}$ of Right Shadow; and at the second Station, suppose the Parts cut be $\left. \begin{matrix} 50 \\ 60 \end{matrix} \right\}$ of Contrary Shadow; and suppose further, that the Distance between the two Stations be $\left. \begin{matrix} 75 \\ 88 \end{matrix} \right\}$ Feet; What is the Height of the Object? Ans. $\left. \begin{matrix} 50 \\ 60 \end{matrix} \right\}$ Feet.

P R O P. IV.

To take the Height of an accessible Object, by the Degrees on the Limb of the Quadrant.

The R U L E.

As the Co-sine of the Degrees, cut by the Thread, (when the Eye through the Sights, sees the Apex of the Object) is to the Sine thereof; so is the Distance between the Station and the Object, to the Height of the Object, above the Horizontal Line of the Eye.

Example.

If the Degrees cut by the Thread be 30, and the Distance from the Station to the Object be 86.6 Feet; What is the height of the Object? Ans. 50 Feet. For, as 8606 (the Co-sine of 30 Degrees,) is to 5000 (the Sine thereof;) so is 86.6 Feet (the Distance from the Station to the Object,) to 50 Feet, the height of the Object.

Note: The distance from the Station to the Object, must be taken according to the Note, in Prop. I.

P R O P. V.

To take the height of an inaccessible Object, by the Degrees on the Limb of the Quadrant.

The R U L E.

This Proposition is performed by 2 Stations; at each of which take a view of the Apex of the Object, thro' the Sights of the Quadrant, and Note the Degrees cut by the Thread; then, from the Degrees cut at the nearest Station to the Object, Subtract the Degrees cut at the farthest Station from the Object, noting the Remainder: Then say, as the sine of this Remainder, is to sine of the Degrees cut at the farthest Station; so is the Distance between the 2 Stations, to a 4th. Then as the Radius, is to this 4th; so is the sine of the Degrees cut at

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at the nearest Station, to the height of the Object above the Horizontal Line of the Eye, at that Station.

Example 1.

If the Degrees (cut by the Thread) at the nearest Station were 45, and at the farthest 30; and the Distance between the two Stations 30 Feet; What is the height of the Object? *Ans.* 40.757 Feet. For, as 2602. (the sine of 15 Degrees, the difference of Degrees cut at each Station,) is to 3000, (the sine of 30 Degrees, cut at the farthest Station, so is 30 Feet (the Distance between the two Stations) to 57.64 Feet (a 4th.) Then again, as 10000 (the Radius,) is to 57.64 (the 4th.) so is 7071 (the sine of 45 Degrees, cut at the nearest Station,) to 40.757 Feet, the Height of the Object.

P. R. O. P. O. VL

Another way to take the height of an inaccessible Object, by the Degrees on the Limb of the Quadrant.

The R. U. L. III

Having taken your first Station, as near the Object as conveniently you may, and made Observation of the Degrees cut (by the Thread) on the Limb of the Quadrant; then for the 2d Station, go back (in a Right-Line with the Object and first Station) so far, that the Degrees cut by the Thread may be but just half as many as were cut at the first Station. Then, as, as the Radius, is to the sine of the Degrees cut at the 1st Station, so is the Distance between the 2 Stations, to the height of the Object above the level of the Eye at the first Station.

Example.

If the Degrees cut at the nearest Station to the Object were 80, and the Distance between the two Stations 100 Feet; What is the height of the Object? *Ans.*

98.48.

98.48 Feet. (i. e. almost $98\frac{1}{2}$ Feet) for, for as 100000 : 9848 :: 100 : 98.48.

Note: In this and the preceding Proposition, you ought to choose as level Ground (for the distance between the Stations) as you can.

PROP. VII.

Another more general way, by any two Stations, taken at Random.

The RULE.

Having made Observation at each Station; then say, as the difference of the Co-Tangents of the Degrees cut at each Station, is to the Distance between the 2 Stations; so is the Radius, to the height of the Object.

Example.

Let the Degrees cut at the nearest Station be 80, and at the farthest Station 40, and the distance between the Stations 150 Feet; What is the height of the Object? *Ans*y. 147.72 Feet. For the Co-Tangent of 80 Degrees is 1763, of 40 Degrees is 11917. their Difference is 10154. Then say, as 10154 : 10000 :: 150 : 147.72.

SECT. II.

Of taking heights by the Projected, or Reflected Image of the Object.

The projected Image of an Object is the Umbra or Shadow of it caused by the Sun, &c. And the Reflected Image is the Icon or Representation of it in Water, &c. Which is also commonly call'd a Shadow.

Now Heights may be taken several ways by the Shadow of the Object; but I shall (in this place) mention only two or three of them.

PROP. I.

To take the height of an accessible Object, by its Shadow made by the Sun or Moon, and the help only of a Line and Plumm.

The

The R U L E.

Say, As the Length of the Shadow of the Line and Plummēt, is to the Length of the Line and Plummēt; so is the Length of the Shadow of the Object, to the Height of the Object above the Ground.

Example.

If the Length of the Line and Plummēt be 5 Feet, and the Length of its Shadow be 3 Feet; and the Length of the Shadow of the Object be 30 Feet; What is the Height of the Object? Ans. 50 Feet. For, As 3 : 5 :: 30 : 50.

Note (1.) That the Length of the Shadow of the Object, should be taken to a Perpendicular under that Point of the Object which gives the longest Shadow; which in Steeples, &c. will be the Middle of the Object; but in such things as are flat on the Top, it will be that side next the Shadow.

(2.) That the Length of the Shadow of the Line and Plummēt, (suspended from the Hand, or the like) ought to be taken on the same Ground on which the Shadow of the Object falls; for so the Acclivity or Declivity of the Ground, will cause no Variation.

(3.) That this Proposition might also be wrought by the help of the Quadrant, as in Prop. 2. of Sect. 1. But the Rule deliver'd in this Proposition is to be prefer'd to that, as being more accurate.

P R O P. II.

To take the Height of an accessible Object, by its Shadow in a Basōn of Water.

The R U L E.

Having set down the Basōn of Water any where before the Object; go back (in a Right Line with the Object and Basōn) till the Shadow of the Apex of the Object appears in the Middle of the Basōn: Then say, As the Distance from the Middle of the Basōn to your Station,

is to the Height of your Eye above the Level of the Basin; so is the Distance from the Object to the Basin, to the Height above the Level of the Basin.

Example.

Let the Distance from the Basin to the Station be 10 Feet, and the Height of the Eye 5 Feet, and the Distance from the Basin to the Object 100 Feet; What is the Height of the Object? *Ans.* 30 Feet. For, As 10 : 5 :: 100 : 30.

Note (1.) That in taking a Height by this Method, you ought to choose such a Station, as that the Space between it and the Object, be as level Ground as may be.

(2.) That the Distance between the Basin and the Object, should be taken to a Perpendicular under that Point of the Object which you saw in the Middle of the Basin. *v.* Note 1. of Prop. I. of Sect. 2.

P R O P. III.

To take the Height of an inaccessible Object, by its Shadow in a Basin of Water.

The R U L E.

Having made one Observation, as in the preceding Proposition; mark the Station of the Basin, and also your own Station. Then remove the Basin backwards, in a Right Line from the Object, (any Distance that you please) and make (a second) Observation as before. Then say, As the Height of your Eye (above the Level of the Basin) at your { first } Station, is to the Distance from the Middle of the Basin to your Station; so is 1, to a 4th. Having thus found a 4th Proportional for each Station; say, As the Difference between these 4ths, is to 1; so is the Distance between the two Stations of the Basin, to the Height of the Object.

Exam-

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Example.

If in the { first } Observation, the Distance between the Bason and your Station be { 5 } Feet, and the Height of the Eye in each Observation 5 Feet, and the Distance between the two Stations of the Bason 150 Feet; What is the Height of the Object? Ans. 75 Feet.

Note. That the first Note in the preceding Proposition, takes place also in this.

So, let this suffice (at this time) for Altimetry, or Taking of Heights. I come now to

S E C T. III.

Of Longimetry, or Taking Distances.

As *Heights*, so also *Distances*, may (as was before hinted) be taken by different Ways, almost innumerable. I shall here mention only some few of the most easy and practicable Ways.

P R O P. I.

To take the lateral Distance of an Object, by the Quadrant of the Quadrant.

The RULE.

Having, at two different Stations, taken a View of the Apex of the Object, through the Sights of the Quadrant, and noted the Divisions cut (by the Thread) at each Station; Then, find the Height of the Object, by Prop. 3. of Sect. I. Then, to find the Distance of the Object from any of the Stations, say, As the { Radius of the Quadrant, is to the Parts of Right Shadow cut; }
Contrary Shadow cut, is to the Radius of the Quadrant; }
so is the Height of the Object, to its Distance.

Example.

If the Radius of the Quadrant be 60, and the Parts of Contrary Shadow cut at the nearest Station (to the Object) be 3, and at the farthest Station (from the Object) 4, and the Distance between the two Stations be 150 Feet; What is the Distance of the Object, from the farther Station? Ans. The Height of the Object will be found (by Prop. 3. of Sect. 1.) to be 50 Feet; and (from thence) its Distance from the farther Station, will be found to be 750 Feet. For, As 4 : 60 :: 50 : 750.

P R O P. II.

To take the lateral Distance of an Object, by the Degrees on the Limb of the Quadrant.

The RULE.

First find (by Prop. 3. of Sect. 1.) the Height of the Object above the Horizon of the nearest Station. Then say, As the Sine of the Degrees cut (by the Thread) at the nearest Station, is to the Co-sine thereof; so is the Height of the Object (now found) to its Distance from the nearest Station.

Example.

If the Degrees cut at the nearest Station be 5, and at the farthest 4, and the Distance between the two Stations be 150 Feet; What is the Distance of the Object, from any of the Stations? Ans. The Height of the Object will be found (by Prop. 3. of Sect. 1.) to be 52.17 Feet; and from thence (by the present Proposition) its Distance from the nearest Station, will be found to be 596 Feet. To which, if you add 150 Feet, the Distance between the two Stations, it makes 746 Feet, for the Distance between the Object and the farthest Station.

P R O P.

P R O P. III.

To take the lateral Distance of an Object, by its Shadow in a Bason of Water.

The RULE.

First find the Height of the Object, by Prop. 3. of Sect. 2. Then say, As the Height of your Eye (in any of your Stations), is to the Distance of your Station from the Bason; so is the Height of the Object (now found) to its Distance from the Bason.

Example.

If in the first Observation, the Distance between the Bason and your Station, be 5 Feet, and in the second Observation 10 Feet; and the Height of the Eye in each Station 5 Feet; What is the Distance of the Object from the farthest Station? Ans. 100 Feet.

I might here shew many more Ways to take Heights and Distances; but at this time let these suffice.

C H A P. III.

Of Planimetry, or the Mensuration of Superficial Figures by the Line.

I Shall here forbear to lay down the Definitions of Geometrical Figures, for the Reason mention'd in the Proeme of this Book. And therefore I shall fall directly to the Matter in Hand; and shew how in

P R O P. I.

To find the Area or Superficiality of any Paralellogram, (whether Right-Angl'd, as the Geometrical Square and

I 2

Oblong;

Oblong; or Oblique-Angl'd, as the Rhomb and Rhomboid;) as also of any Right-lin'd Triangle, (whether Equilateral, Ifoſcle, or Scalene;) and alſo of a Trapezia.

These are all the Right-lin'd plain Figures in Geometry, (except Polygons;) of which hereafter; I ſhall therefore for theſe, lay down a brief Table of Centres or Points on the Line, by which and the following Rules, you may find their Area, in any uſual Meaſure; as Inches, Feet, Yards, Squares, Perches, or Miles.

Breadth given in	Length given in	Area re- quir'd in	The Centre is at
Any Meaſure	the ſame.	the ſame.	1.
		Feet	144.
		Yards	1296.
		Squares	14400.
		Perches	39204.
		Acres	6272640.
		Miles	4014489600.
	Inches.	Feet	12.
		Yards	108.
		Squares	1200.
		Perches	3267.
		Acres	522720.
		Miles	334540800.
	Feet.	Yards	36.
		Squares	400.
		Perches	1089.
		Acres	174240.
		Miles	111513600.
	Yards.	Perches	198.
		Acres	31680.
		Miles	20275200.
	Perches.	Miles	63360.
	Miles.		

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A Continuation of the preceding Table.

Breadth given in	Length given in	Area requir'd in	The Centre is at
Feet.	Feet.	Yards	9.1028
		Squares	100.
		Perches	272.25
		Acres	43560.
		Miles	27878400.
	Yards.	Yards	3.
		Squares	33.333, &c.
		Perches	90.75
		Acres	14520.
		Miles	9292800.
Yards.	Perches.	Perches	16.5.
		Acres	2640.
		Miles	1689478.78
	Miles.	Miles	5280.
		Squares	11.1111, &c.
	Yards.	Perches	30.25
		Acres	4840.
		Miles	3097600.
	Perches.	Perches	5.5
		Acres	880.
Perches.	Miles.	Miles	563200.
		Miles	1760.
		Acres	160.
	Perches.	Miles	102400.
		Miles	320.

The Use of the preceding Table.

When a Superficies is to be measur'd, consider what Measures the Dimensions are given in, and in what Measure the Area is requir'd; and accordingly enter the Table; finding in the first Column the Denominative Measure of the given Breadth; and (within that Line

of Connection) in the second Column, find the Denominative Measure of the given Length; and (within that Line of Connection) in the third Column, find the Denominative Measure of the requir'd Area; and against it in the fourth Column is a Number, which I call the Centre; because, if at that Number on the Line, a Point or Centre be made, it will be of constant Use, in solving all Questions that have the same *Data* and *Quæstum*. The Number which I call the Centre, being thus found; this is the

General R U L E.

As that Tabular Number, is to the given Breadth; so is the given Length, to the Area sought.

Therefore,

Always extend the Compasses, from the Centre to the given Breadth; the same Extent of the Compasses will reach (being apply'd the same way) from the given Length, to the Area requir'd.

Example.

Let it be requir'd to measure a Rectangl'd Oblong Parallelogram, (as a Plank, Board, or the like,) whose (given) Breadth is 10 Inches, and Length 18 Feet; and whose Area is requir'd in Feet.

Here (because the Breadth is given in Inches) I find Inches in the first Column of the Table; and (within that Line of Connection) in the second Column (because the Length is given in Feet) I find Feet; and (within that Line of Connection) in the third Column (because the Area is requir'd in Feet) I find Feet; and against it in the fourth Column I find 12, which is now the Centre; and is always so when the *Data* and *Quæstum* is the same as it is now. Therefore (by the General Rule above,) I say, As 12 is to the Breadth in Inches; so is the Length in Feet, to the Area in Feet.

So, in this Example, the Extent from 12 (the Centre) to 10, (the Breadth in Inches) will reach the same way from 18 (the Length in Feet) to 15, the Area in Feet requir'd.

So,

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So, let what has been said (together with the following Notes,) suffice for the Explanation and Use of the Table; for it being so plain in it self, I think it needs no farther Exemplification.

Note 1. If the Superficies to be measur'd, be a Geometrical Square; any Side may be accounted the Breadth, and the other the Length.

2. If it be an Oblong, the Breadth and Length are easily known.

3. If it be a Rhomb; one side is accounted the Length, and a Perpendicular let fall from one Angle to that side, is the Breadth.

4. If it be a Rhomboid; one of the longest Sides is the Length, and a Perpendicular (as in the Rhomb) is the Breadth.

5. If it be a Triangle, (of what kind soever,) the Base is the Length, and half the Perpendicular is the Breadth.

6. If it be a Trapezia; the longest Diagonal is the Length, and half the Sum of the Perpendiculars, (let fall from the other Angles to that Diagonal,) is the Breadth.

7. I might here shew how the Circle, and any Regular Polygon, might be measur'd by the preceding Table; (viz. thus; For the Circle account half the Diameter for the Breadth, and half the Circumference for the Length. And, for a Regular Polygon, account the *Linea Pendens* (which is a Perpendicular let fall from the Centre to the middle of one side,) for the Breadth, and half the Circumference for the Length.) But for the Circle, and some Regular Polygons, I shall exhibit other Rules in the following Propositions.

Thus, in this first Proposition, I have compriz'd 50 for each Figure therein mention'd; and they being 8 in Number, (viz. the Geometrical Square, the Oblong, the Rhomb, and Rhomboid, Triangle, Trapezia, Circle, and Polygon,) make in the whole 400 several Propo-

Propositions; each of which are (I think) as plain and intelligible, as if they had been all handl'd distinctly.

PROP. II.

To find (in any usual Measure) what Length any Parallelogram must be, to make a Superficial Foot, Yard, Perch, &c. the Breadth being given in any (usual) Measure.

The RULE.

Enter the Table in Prop. 1. finding in the first Column the Denominative Measure of the given Breadth; and (within that Line of Connection) in the second Column, find the Denominative Measure of the requir'd Length; and (within that Line of Connection) in the third Column, find the Denomination of that Superficial Measure whose Length is requir'd; and against it in the fourth Column, is a Number, which I shall call the Tabular Number: Which Number being thus found; this is the

General RULE.

As the given Breadth, is to the Tabular Number; so is 1, to the Length sought.

Example.

Let there be Parallelogram, whose (given) Breadth is 12 Inches; and let it be requir'd to find the Length in Inches to make a Superficial Yard.

Here (because the Breadth is given in Inches) I find Inches in the first Column of the Table; and (following the Rule) in the second Column, (because the Length is requir'd in Inches) I find Inches; and in the third Column (because the Superficial Measure, whose Length is requir'd, is a Yard,) I find Yards; and against it, in the fourth Column, I find 1296. Which is the Tabular Number for this Question; and for all others of the same Data and Question. Therefore (by the General Rule above) I say, As 12 (the given Breadth),

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is to 1296, (the Tabular Number); 'o is 1, to 108, the Length in Inches (to make a Superficial Yard) requir'd.

Again, If from the same *Data*, the Length had been requir'd in Feet; the Tabular Number (by the same Method) would be found to be 108; and from thence, the Length requir'd, would be found to be 9 Feet.

Again, If the Length (from the same *Data*) be requir'd in Yards; the Tabular Number will be found to be 36, and the Length requir'd 3 Yards.

This Proposition being so very plain, (and I affecting Brevity) I forbear to add any more Examples.

Note. If this Proposition be (as it may) apply'd to all the Figures mention'd in Prop. 1. then here will also be 400 Problems, and all resolvable from the Table in Prop. 1. and the Rules above.

P R O P. III.

To find the Area or Superficiality of a Circle.

As { 14, to 11; }
 { 1.27324, to 1; } so is the Square of the Diameter,

ter, to the Area, according to { Archimedes.
 Van Culen.
 Adrianus Metius.

Otherwise,

As { 7, to 22; }
 { 113, to 355; } so is the Square of the Semi-diameter, to the Area, according to { Archimedes.
 Adrianus Metius.
 Van Culen.

Otherwise,

As { 88, to 7; }
 { 1, to .07958; } so is the Square of the Circumference, to the Area.

Other.

Otherwise,

As 1, is to the Semi-Diameter; so is the Semi-Circumference, to the Area.

Otherwise,

As 4, is to the Diameter; so is the Circumference, to the Area.

Or more briefly, thus;

Always extend the Compasses from 1, to the (given) Diameter of the Circle; the same Extent being turn'd

twice (the same way) from $\left. \begin{array}{l} .7857 \\ .7853 \\ .7854 \end{array} \right\}$ will reach to

the Area of the Circle; according to $\left\{ \begin{array}{l} \text{Archimedes,} \\ \text{Adri. Metius,} \\ \text{Van Culen.} \end{array} \right.$

Note. The Unit (in this Case) is best taken at 10, in the Middle of the Line; for so .785, &c. will be at 7.85, &c. on the first Part of the Line.

Example.

If the Diameter of a Circle be 10; What is the Area

thereof? Ans. $\left\{ \begin{array}{l} 78.57 \\ 78.53 \\ 78.54 \end{array} \right\}$ according to $\left\{ \begin{array}{l} \text{Archimedes,} \\ \text{Adri. Metius,} \\ \text{Van Culen.} \end{array} \right.$

PROP. IV.

To find the Area or Superficiality of the Sector of a Circle.

The RULE.

As 1, is to the Semi-Diameter of the Circle of which it is a Sector; so is the Semi-Circumference of the Circular Part, to the Area of the Sector, sought.

Example.

If the Semi-Diameter of the Circle be 7, and the Semi-Circumference of the Circular Part be 8; what is the Area of that Sector? Ans. 56.

PROP.

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PROP. V.

To find the Area or Superficiality of the Segment of a Circle.

The RULE.

(1.) Measure it (by Prop. 4.) as the Sector of a Circle, only if the Segment be $\left\{ \begin{array}{l} \text{lesser} \\ \text{greater} \end{array} \right\}$ than a Semi-

Circle, $\left\{ \begin{array}{l} \text{Subtract from} \\ \text{add to} \end{array} \right\}$ the Area so found, the Area of an Isosceles Triangle, whose Base is the Chord, and Perpendic. the Excess of the $\left\{ \begin{array}{l} \text{Versed Sine above the Radius.} \\ \text{Radius above the Versed Sine.} \end{array} \right\}$

(2.) But, the common Data for measuring the Segment of a Circle, being the Chord and Versed Sine; before the Sector can be measured, the Radius (or semi-Diameter) and $\frac{1}{2}$ the Circumference of the Arch, must be found; (they being the common Data for measuring a Sector;) thus,

(3.) To find the Radius, say, As the Versed Sine, is to the Square of $\frac{1}{2}$ the Chord; so is 1 to a 4th, which added to the Versed Sine, gives the whole Diameter; half of, which is the Radius.

(4.) To find half the Circumference of the Arch of the Segments, this is

The RULE.

If the Segment be $\left\{ \begin{array}{l} \text{lesser} \\ \text{greater} \end{array} \right\}$ than a Semi-Circle, say, as the Radius added to half the Excess of the $\left\{ \begin{array}{l} \text{Radius above the Versed Sine,} \\ \text{Versed Sine above the Radius,} \end{array} \right\}$ is to half the Chord; so is 86, to the $\left\{ \begin{array}{l} \text{Complement to 90 of the} \\ \text{half Circumference of the Arch, in Degrees.} \end{array} \right\}$

(5.) Then, (the Diameter being found as above) find the Circumference of the whole Circle, (by Prop. 1. of

1. of Chap. 1.) Then say, as 360, to the whole Circumference (now found;) so is the half Circumference of the Arch in Degrees, (found as above,) to the same, in such Measures as the Chord and Versed Sine were given in.

So, things being thus far prepar'd; the Sector is measur'd by Prop. 4. and the Segment by the 1 Part of the Rule in this Proposition.

Example.

Let there be the Segment of a Circle to be Measur'd, whole Chord is 12 Inches; What is the Area of this Segment? Answ. 32.97 Inches. For the Radius is found (by the third part of the Rule) to be 13 Inches; and the half Circumference of the Arch, (by the 4th and 5th Part of the Rule) is found to be 66.9 Inches, whence (by the 4th Prop.) the Area of the Sector is 86.97 Inches, and from thence (by the 1st. Part of the Rule of this Prop.) the Area of the Segment is found to be 32.97 Inches, as was said above.

Note: (1) If the Versed Sine be $\left\{ \begin{array}{l} \text{lesser} \\ \text{greater} \end{array} \right\}$ than the Chord; then the Segment is $\left\{ \begin{array}{l} \text{lesser} \\ \text{greater} \end{array} \right\}$ than the Semi-Circle.

(2.) If the Versed Sine be equal to half the Chord; then the Segment is just half the Circle; and so is easy enough measured.

P R O P. VI.

To find the Area or Superficiality of an Ellipsis.

The R U L E.

As $\left\{ \begin{array}{l} 14 \text{ to } 11; \\ 1.27324 \text{ to } 1; \\ 452 \text{ to } 355; \end{array} \right\}$ so is the Rectangle or Fact of the Transverse and conjugate Diameters, to the Area of the Ellipsis; according to $\left\{ \begin{array}{l} \text{Archimedes.} \\ \text{Van Culen.} \\ \text{Adrianus Metius.} \end{array} \right\}$

Ex-

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Example. O R P

If the Transverse Diameter of an Ellipsis be 12.4 Inches, and the Conjugate Diameter be 8 Inches; What

is the Area of that Ellipsis? Ans. $\left. \begin{matrix} 78.57 \\ 78.52 \\ 78.44 \end{matrix} \right\}$ Inches.

according to $\left\{ \begin{matrix} \text{Archimedes.} \\ \text{Adri Metim.} \\ \text{Van Culen.} \end{matrix} \right.$

P R O P. VII.

To find the Area or Superficiality of any Regular Polygon; from the Trigon to the Duodecagon.

The R U L E.

Always extend the Compasses from 1, to the Side of the given Polygon; the same Extent of the Compasses being turn'd twice (the same way) from

0.433	Trigon.
1. ---	Tetragon.
1.7204	Pentagon.
2.5980	Hexagon.
3.6397	Heptagon.
4.8284	Octagon.
6.18183	Nonagon.
7.69435	Decagon.
9.36386	Endecagon.
11.19648	Duodecagon.

Will reach to the Area of the

For, As 1, is to the Square of the Side of any Regular Polygon; so are the Numbers in the Table above, to the Area of their respective Polygons.

Example.

If the Side of a Pentagon be 10; What is its Area? Ans. 172.04. For the Extent from 1, to 10, will reach (being turn'd twice the same way) from 1.7204, to 172.04.

K

P R O P.

PROB. VII.

To find the Area or Superficiality of any Irregular Polygon.

THE RULE.

Reduce the Irregular Polygon (by drawing Lines from Angle to Angle) either all into Triangles; or into one or more Trapezia's, and the rest into Triangles. Then measure those Trapezia's, or Triangles; by Prop. I. and add them all together; so have you the Area of the Irregular Polygon.

Otherwise,

Delineate your Figure (to be measur'd) upon Paper; and by the same Scale draw also a Tetragon upon the same Paper; then cutting them out by the Lines that terminate their Figure; weigh them severally in a Pair of Gold Scales. Then say, As the Weight of the Tetragon, is to his Area; so is the Weight of the Irregular Polygon, to his Area.

CHAP. IV.

Of Plano-Stereometry, or the Mensuration of the Superficiality of Solid Bodies.

Solid Bodies (whose Superficies there may be occasion to measure) are, either the 5 Platonic Bodies, or others; of which the former are absolutely Regular; and some of the others are Regular in their Kind: of which there is the Globe, the Cylinder, the Cone, the Prism, and the Pyramid, &c. All which may be dichotomiz'd; and of some of them there are divers Kinds.

But the Superficiality of Solid Bodies being not so useful as their Solidity; I shall not insist so much upon it; but shall only (in this place) shew how to find the Superficiality of those 5 Solids above mention'd, and of the 5 Platonic Bodies.

PROB.

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PROP. I.

To find the Superficiality of a Globe.

The **R U L E.**

As $\left\{ \begin{array}{l} 7 \text{ to } 22 \\ 1 \text{ to } 3.14159 \end{array} \right\}$ is to the Square of the Diameter, to the Superficiality requir'd, according to
Archimedes.
Van Culen.
Adrian. Metius.

Otherwise,
 Always extend the Compasses from 1 to the Diameter of the Globe; then that Extent being turn'd twice (the same way) from $\left\{ \begin{array}{l} 3.1428571 \\ 3.1415926 \\ 3.1415929 \end{array} \right\}$ will reach to the Superficiality requir'd, according to

Archimedes.
Van Culen.
Adrian. Metius.
Example.

If the Diameter of a Globe be 10. What is its Superficiality? Ans. (according to $\left\{ \begin{array}{l} \text{Archimedes} \quad 314.28571 \\ \text{Van. Culen} \quad 314.15926 \\ \text{Adrian. Metius} \quad 314.15929 \end{array} \right\}$)

PROP. II.

To find the Superficiality of the Segment of a Globe.

The **R U L E.**

First find the Chord of $\frac{1}{2}$ the Segment, thus: To the Square of half the Versed Sine, add the Square of half the Chord of the given Segment; the Square Root of the Sum, is the Chord of half the given Segment. Then,

Say, (by Duplicate Proportion Direct, v. Book I. Chap. 15. Sect. 1.) As 1, is to the Chord of half the

Segment; so is $\left\{ \begin{array}{l} 3.1415926 \\ 3.1415929 \end{array} \right\}$ to the Superficiality of

the Convexity, (according to $\left\{ \begin{array}{l} \text{Archimedes} \\ \text{Van Culen} \\ \text{Adri. Metius} \end{array} \right\}$) to

which add the Superficiality of the Circular Base, (found by Chap. 3. Prop. 3.) and you have the Superficiality of the whole Segment requir'd.

Example.

Let the Chord of the Segment of a Globe be 8, and its Versed Sine 3; What is the Superficiality thereof?

Ans. (according to *Metius*) 78.5398295.

P R O P. III.

To find the Superficiality of a Cylinder.

The R U L E.

As 1, is to the Length, so is the Circumference (of the Cylinder) to the Superficiality of the Convexity; to which add the Area's of the Circles at each End, (found by Chap. 3. Prop. 3.) and you have the Superficiality of the Cylinder requir'd.

Example. If the Length of a Cylinder be 40, and its Circumference 22; What is the Superficiality thereof? Ans. 957.

P R O P. IV.

To find the Superficiality of a Cone.

The R U L E.

As 1, is to the Oblique (not Perpendicular) A'titude of the Cone; so is half the Circumference of the Base, to the Superficiality of the Convexity; to which add, the Area of the Circular Base, (found by Chap. 3. Prop. 3.) and you have the whole Superficiality of the Cone requir'd.

Ex-

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Example.

If the Oblique Altitude of the Cone be 40; and the Circumference of the Base 44; What is the Superficiality thereof? Ans. 1254.

PROP. V.

To find the Superficiality of the Frustum of a Cone.

The RULE.

As 1, is to the Oblique Altitude of the Frustum; so is half the Sum of the Circumference at the Top and Bottom, to the Superficiality of the Convexity; to which add the Area of the Circular Top and Base, (found by Chap. 3. Prop. 3.) and you have the whole Superficiality of the Frustum required.

Example.

If the Oblique Altitude of the Frustum be 20, and the Circumference of the Top 22, and of the Base 44; What is the Superficiality thereof? Ans. 852.

PROP. VI.

To find the Superficiality of a Prism.

The RULE.

The Superficiality of a Prism, may be found in all respects like that of a Cylinder, (V. S. Prop. 3.) respect only being had to the Figure of its Bases; which (whether they are Trigons, or Polygons, or, &c.) may be measured, either by Prop. 1. or 6. or 7. or 8. of Chap. 3.

PROP. VII.

To find the Superficiality of a Pyramid.

The RULE.

The Superficiality of a Pyramid, may be found as that of a Cone (v. s. Prop. 4.) respect only being had to its Base, which may be measured, either by Prop. 1. or 6. or 7. or 8. of Chap. 3.

PROP. VIII.

To find the Superficiality of the Frustum of a Pyramid.

The RULE.

The Superficiality of the Frustum of a Pyramid, may be found as that of the Frustum of a Cone, (v. s. Prop. 5.) respect had to its Bases; which are measur'd as in the preceding Proposition.

PROP. IX.

To find the Superficiality of any of the 5 Platonic Bodies.

The RULE.

Say, (by Duplicate Proportion Direct; v. Book 1. Chap. 15. Sect. 1.) As 1 is to the side of the given Platonic Body; so

is	{	1.73205	{	the Superficiality of the	Tetraëdron.
		3.46410			Octaëdron.
		6.			Hexaëdron.
		8.66025			Icosaëdron.
		20.64573			Dodecaëdron.

Or, As 1, is to the Square of the side, of any of the Platonic Bodies; so are the Numbers above in this Proposition, to the Superficiality of their respective Platonic Body.

Example 1.

If the Side of a Tetraëdron be 10; What is its Superficiality? Answ. 173.205.

Thus (kind Reader) thou hast 828 Propositions for the Mensuration of Superficies, viz. 815 in Chap. 3, and 13. in this.

C H A P.

CHAP. V.

Of Steriometry, or the Mensuration of Solids;
by the Line.

THe most usual Solids to be Measur'd, are the Quadrangular Prism or Parallelopipedon, and the Cylinder. I shall therefore begin with these, shewing how in

PROP. I.

To find the Solidity of a Parallelopipedon (as also of a Cylinder;) in all the most usual Measures, and from all the most usual Data.

To avoid Tediouſity, I here exhibit a brief Table of Centres or Points on the Line, by which and the following Rules, all Questions relating to this Proposition, may be easily and speedily resolved.

The TABLE.

Diameter or side gi- ven in	Length given, in	Solidity requ'r'd in	Centre for Square is at	Centre for Round is at
Any Measure.	the same	the same	1. —	1.128
Inches	Inch. {	Feet.	41.569	46.869
		Yards.	216. —	243.68
	Feet {	Feet.	12. —	13.537
		Yards.	62.353	70.344
Feet	Yards.	Yard.	36. —	40.604
	Feet.	Yards.	5 196	5.862
	Yards.	Yards.	3. —	3.384

The Use of the Preceding Table.

(1.) When either a Square Prism, or a Cylinder is to be Measur'd; consider in what Measures the Dimensions

tions are given, and in what Measures the Solidity is requir'd; and accordingly enter the Table; finding in the first Column the Denominative Measure of the given Diameter of the Cylinder, or side of the Parallelopipedon; and (within that Line of Connection) in the second Column, find the Denominative Measure of the given length; and (within that Line of Connection) in the third Column, find the Denominative Measure of the requir'd Solidity; and against it in the fourth Column, (for a Parallelopipedon) or in the fifth Column, (for a Cylinder) you will find a Number, which I call the Centre; because if at that Number on the Line, a Point or-Centre be made, it will be of constant use, in all Questions of the same Data and Question. The Centre being thus found; this is the

General Rule.

Say, (by Duplicate Proportion Direct, V. Book I. Chap. 15. Sect. 1.) As the Centre is to the given Diameter of the Cylinder, or Side of the Parallelopipedon; so is the length to the Solidity requir'd.

Therefore,

Always extend the Compasses from the Centre, to the given Diameter of the Cylinder, or side of the Parallelopipedon; the same Extent being turn'd twice (the same way) from the given length, will reach to the Solidity requir'd.

Example 1.

(1.) Of a Parallelopipedon to be Measured; whose given length I suppose 16 Feet, and the side of the Square its Base 18 Inches, and whose Solidity is required in Feet.

Here, because the side of the Parallelopipedon is given in Inches, I find Inches in the first Column of the Table; and (within that Line of Connection) in the second Column (because the length is given in Feet) I find Feet; and within that Line of Connection, in the third

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third Column (because the Solidity is requir'd in Feet) I find Feet; and against it in the fourth Column I find 12. which is now the Centre, (and is so in all Questions of the same *Data* and *Questum*.) Therefore, (by the General Rule above) I extend the Compasses from 12 (the Centre) to 18 (the side of the Parallelopipedon in Inches,) and turning the same Extent of the Compasses twice the same way, from 15, (the length in Feet,) they will reach to 36 the Solidity in Feet, requir'd.

(2.) But if the Base of the Parallelopipedon be (not a Geometrical Square,) but an Oblong; then, the customary way, is to add together the breadth and thickness, and take half their Sum for the side of a Mean Square; and then to Measure it as in the Preceding Example. But this way (tho' generally us'd by all Timber Measures) is false and very erroneous; and that more or less, according to the greater or lesser Difference of the breadth and thickness.

But the true way to measure unequal sided Parallelopipedons is thus; viz. Either (1.) Divide the space (on the Line) between the breadth and thickness, into two equal parts, and that middle Point shews the true side of a Mean Square: and then Measure the Parallelopipedon, as in the preceding Example. Or, (secondly, which will effect the same thing,) Extend the Compasses from the Centre, to the $\left\{ \begin{array}{l} \text{Breadth,} \\ \text{Thickness,} \end{array} \right\}$ the same Extent will reach the same way from the $\left\{ \begin{array}{l} \text{Thickness,} \\ \text{Breadth,} \end{array} \right\}$ to a fourth. Then the Extent from the Centre, to that fourth, will reach from the length, to the Solidity, requir'd.

Example 2.

Suppose a Parallelopipedon, to be Measur'd, was 24 Inches broad, and 12 Inches thick, and 18 Feet long; and the Solidity requir'd in Feet. Here, by the Customary way, I add 24 Inches the Breadth, to 12 Inches the Thickness, their Sum is 36, the half whereof is 18 Inches;

Inches; (which is taken to be the side of a Square equal to the Oblong, and the Parallelopipedon is supos'd to be thus reduced from an Oblong, to a square Prism) then I measure it as in Example 1. And find the solidity to be 40.5 Solid Feet; which is 4.5 Feet more than the true solidity, as will appear from the true measuring; viz. thus, I divide the space on the Line between 24 (the breadth) and 12 (the Thickness) into two equal parts, and find the Compasses to rest at 17 Fere, which is the true side of a square equal to the Oblong Base of the Parallelopipedon; then I measure it as in Example 1 of this Proposition, and find the Solidity to be 36 Feet. The same will be found, if (by the second true way) I extend the Compasses from 12 (the Centre) to 24 (the breadth) the same Extent will reach from 12 (the Thickness) to a fourth, which here will be 24, then the Extent from 12 (the Centre) to (this fourth, viz. 24) will reach from 18 (the Length in Feet,) to 36, the Solidity true in Feet, as before.

Example 2.

(3.) Of Measuring a Cylinder, whose given Diameter, suppose 18 Inches, and Length 18 Feet long; and whose Solidity is required in Feet. Here I find the Centre as before for the Parallelopipedon, with this difference only; that whereas then I found it in the fourth, now I find it in the fifth Column; (v. sc. in Rule 1. of this Prop.) Thus the Centre (in this Example,) will be found to be at 12.537.

The Centre being thus found, I proceed to measure the Cylinder (by the General Rule above in this Prop.) and find its Solidity to be 31.83 Feet.

(4.) But, if the Base of the Cylinder (to be measured, be an Ellipsis; find a mean Proportional between the two Diameters of the Ellipsis, as you did before for the Oblong Base of the Parallelopipedon; viz. Divide the space (on the Line) between the Transvers and conjugate Diameters (of the Ellipsis) into two equal parts. So having thus found the Diameter of a Circle equal to the Ellipsis

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Ellipsis, proceed to Measure the Cylinder, as in Example 3.

Otherwise, extend the Compasses from the Centre, to the $\left\{ \begin{array}{l} \text{Conjugate} \\ \text{Transvers} \end{array} \right\}$ Diameter, the same Extent will reach (the same way) from the $\left\{ \begin{array}{l} \text{Conjugate} \\ \text{Transvers} \end{array} \right\}$ Diameter, to a fourth: then again, the Extent from the Centre, to this fourth, will reach (the same way) from the length, to the Solidity requir'd.

Example 4.

Suppose an Elliptical Cylinder (for so I may call such a Figure as I have now been speaking of) were to be measured, the Transverse Diameter of its Bale is 24 Inches, and its conjugate Diameter 12 Inches, and the length of the Cylinder 18 Feet, and the Solidity required in Feet. Here by either of the two ways (of measuring an Elliptical Cylinder) mention'd above, the Solidity will be found to be 28.28 Feet.

These Examples, I think are sufficient to make this first Proposition plain and intelligible to any (though but an Ordinary) Capacity.

And thus thou hast 20 Propositions in this one, viz. 10 for the Parallelopipedon; and 10 for the Cylinder, (and if we include the Elliptical Cylinder, and unequal sided Parallelopipedon, there will be 40 Propositions included in this one,) for though there be but 2 Centres (for each) in the Table; yet the first Centre serves for 3 (or more) different Cases, viz. when the Data and Quesitum are all the same, whither they be Inches, Feet, or Yards, or, &c.

P R O P. II.

To find (in any usual Measure) what length a Parallelopipedon, or Cylinder, must be, to make a Solid Inch, Foot, Yard, &c. the side of the Parallelopipedon, or Diameter of the Cylinder being given, in any usual Measure.

Here are the same Number of Problems as in the preceding Proposition; and all of them solvable from the Table there, and the following

RULES.

R U L E S.

Find (in the first Column of the Table) the Denominative Measure of the given side of the Parallelopipedon, or Diameter of the Cylinder; and (within that Line of Connection) in the second Column, find the Denominative Measure of the required length; and (within that Line of Connection) in the third Column, find the Denomination of that solid Measure whose length is requir'd; and against it in the fourth or fifth Column respectively, is a Number, which I call the Tabular Number: which Number being thus found, this is the

General Rule.

Extend the Compasses from the given side of the Parallelopipedon (or Diameter of the Cylinder) to the Tabular Number; the same Extent being turn'd twice (the same way) from 1, will reach to the length sought.

Example 1.

Let there be a Parallelopipedon, whose side is given 6 Inches, and let it be requir'd to find the length (in Feet) to make a solid Foot. Here entering the Table according to the Rule, I find the Tabular Number to be 19, whence (by the General Rule above) I find, that 4 Feet in length will make a solid Foot.

Example 2.

If (from the same Data) the Length had been requir'd in Inches; the Tabular Number would have been found to be 41.569, and from thence the length in Inches to make a solid Foot, would be found 48.

Example { 3.
4.

If from the same Data, in a Cylinder, it be requir'd to find the length in Feet or Inches, to make a solid Foot, the Tabular Number; in the first Case, will be 13.537 and

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and in the second Case, 46,806; and from thence the Length in Feet will be found to be 3,089, and in Inches, 61,089. And this is the true Length (either in Feet, or Inches,) to make a Solid Foot, as was requir'd.

But if the Base of the Parallelopipedon be an Ob-long, or of the Cylinder be an Ellipsis; Find the Side of a Mean Square, for the Parallelopipedon; and Diameter of a Mean Circle, for the Cylinder; (by the 2d. and 4th Rules of Prop. 1.) and then proceed as before.

P R O P. III

To find the Solidity of a Cylinder, the Circumference being given.

The R U L E.

Find the Diameter, by Chap. 1. Prop. 3. And then proceed to measure the Cylinder, by Prop. 1. of this Chapter. This is the true way of measuring the Cylinder: But,

The customary way is, to take a quarter of the Circumference, for the Side of a Square equal to the Circle; and then to measure it as a Parallelopipedon, by the Rules in Prop. 1. of this Chapter. But this customary way always gives less than the Truth, by almost $\frac{1}{4}$ part of the true content.

P R O P. IV.

To find the Solidity of a Cone.

The R U L E.

First find the Area of its Base, (by Prop. 3. of Chap. 3. or by Prop. 1. Note 7. of the same Chapter.) Then say,

As 3, is to the Area of the Base, (in any Measure); so is the Perpendicular Altitude of the Cone (in the same Measure), to its Solidity, in the same Measure.

Example.

If the Diameter of (the Base of) a Cone be 10, and its Altitude 12; What is the Solidity thereof? Ans. 314.28

Note. If the Axis, or Perpendicular Altitude of a Cone, be not given; it may be found from the Oblique Altitude, thus:

From the Square of the Oblique Altitude, subtract the Square of the Semidiameter of the Base; the Remainder is the Square of the Axis, or Perpendicular Altitude of the Cone requir'd.

P R O P. V.

To find the Solidity of a compressed Cone.

If the Base of a Cone be an Ellipsis; (such a Figure I may call a compressed Cone.) Find a Mean Diameter, as in Rule 4. of Prop. 1. of this Chapter; and then proceed to Measure the Cone by the preceding Proposition. This needs no Example.

P R O P. VI.

To find the Solidity of the Frustum of a Cone.

The R U L E.

First find the Area of the Bases, (by Chap. 3. Prop. 1. Note 7. or by Prop. 3. of the same Chapter.) Then multiply the Area's of the Bases into each other, and (by Book I. Chap. XI.) extract the Square Root of the Product: Add this Root to the Sum of the Area's of the Bases, and multiply this last Sum by $\frac{1}{3}$ of the Perpendicular Altitude of the Frustum; the Product is the true Solidity of the Frustum.

Example.

Let the Diameter of the greater Base be 14, and of the lesser Base 7, and the Perpendicular Altitude 24; What is the Solidity of the Frustum? Ans. 2156.
For the Area of the greater Base is 154, and of the lesser

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less 38.5, their Product is 5929.5 whose square Root is 77. Which added to 192.5 (the Sum of the Area's of the Bases) the Sum is 269.5 Which multiply'd by $8\left(\frac{1}{3}\right)$ of the Altitude) gives 2156, the true Solidity of the Frustum.

P R O P. VII.

To find the Solidity of a Pyramid.

The R U L E.

Consider the Figure of its Base, (whether Trigon, Tetragon, Pentagon, or, &c.) and find the Area thereof, by Prop. 7. of Chap. 3. or by Prop. 1. Note 7. of the same Chapter. The Area of the Base being thus found; say,

As 3 is to the Area of the Base; so is the Perpendicular Altitude, to the Solidity requir'd.

Example.

Let the Side of the Base of a Pentagonal Pyramid be 10, and its Perpendicular Altitude 30; What is the Solidity of this Pyramid? Ans. 1720.4.

P R O P. VIII.

To find the Solidity of the Frustum of a Pyramid.

The R U L E.

Have respect to the Figure of its Base, (whether Trigon, Tetragon, or, &c.) and work in all respects as you did for the Frustum of a Cone, in Prop. 6.

P R O P. IX.

To find the Solidity of a Globe.

The R U L E.

Always extend the Compasses from 1. to the given Diameter; and that Extent being turn'd three times (the same way) from 523809 will reach to the Solidity requir'd.

Example.

Let the Diameter of a Globe (whose Solidity is requir'd) be 12. The Extent from 1 to 12, being turn'd three times (the same way) from .523809, will reach to 905.143, the Solidity of the Globe requir'd.

P R O P. X.

To find the Solidity of the Segment of a Globe.

The R U L E.

First, (if it be not given) Find the Diameter of that Globe, of which the Segment to be measur'd is a part;

Say, As the Altitude of the given Segment, is to 1; so is the Square of the Semidiameter of the Base (of the given Segment) added to the Square of the Altitude, to the Diameter of the Globe requir'd: from which subtract the Altitude of the given Segment; the Remainder is the Altitude of the other Segment: Or, the Altitude of the other Segment may be found more speedily, thus; Say,

As the Altitude of the given Segment, is to the Square of the Semidiameter of its Base; so is 1, to the Altitude of the other Segment requir'd. Which being thus found; Then,

Secondly, Say, As 1, is to the Altitude of the given Segment; so is Square of the Diameter of its Base, to a 4th. Then,

Thirdly, Say, As the Altitude of the other Segment, is to $\frac{1}{2}$ it self, added to $\frac{1}{2}$ of the given Segment's Altitude; so is the 4th (last found), to a 5th. Then,

Fourthly, Say, As 14, 11; so is the 5th (last found), to the Solidity of the Segment requir'd.

Example.
If the Altitude of a Segment of a Globe be 4, and the Diameter of its Base 12; What is the Solidity thereof? Ans. 260.07.

P R O P.

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PROP. XI.

Another way to find the Solidity of the Segment of a Globe.

The RULE.

First (by the first Part of the Rule in Prop. 10.) find the Diameter of the Globe of which the Segment to be measur'd is a part. Then,

Secondly, From the Diameter of the Globe, subtract the Frustum's Altitude, and to the Remainder add the Globe's Semidiameter, noting the Sum. Then,

Thirdly, say, As 1, is to the Sum, so is the Square of the Frustum's Altitude, to a 4th.

Fourthly, say, As 1 is to 1.0488, so is the 4th (last found), to the Solidity of the Frustum or Segment requir'd.

PROP. XII.

Another more easy and more expeditious Way, to find the Solidity of the Segment of a Globe.

The RULE.

As 1, is to the Diameter of the Segment of a Globe; so is half its Altitude (twice turn'd on the Line), to its Solidity, near.

Not. This Rule is not Mathematically exact; but it is very easy and expeditious, and may serve well enough for Common Practice.

PROP. XIII.

To find the Solidity of a Prism.

The RULE.

Consider the Figure of its Base, (whether Triangle, Tetragon, Oblong, Pentagon, or, &c.) and find the Area thereof, by Prop. 7. of Chap. 3. or by Prop. 1. Note 1, 2, 3, 4, 5, 6, 7, 8, 9, of the same Chapter. Then say, As 1, is to the Area of the Base, so is the Length of the Prism, to his Solidity.

Example.

If the Side of a Pentagonal Prism be 10, and its Length 100, What is the Solidity thereof? *Ans.* 17204. For the Area of the Base is found (by Prop. 7. of Chap. 3.) to be 172.04. Then, As 1 : 172.04 :: 100 : 17204.

PROP. XIV.

To find the Solidity of a Spheroid.

The RULE. The First's Altitude, and the Transverse Diameter, and the Conjugate Diameter, and the Solidity of the Spheroid.

As 14 to 11, so is 127224 to 1; so is the Square of the Conjugate Diameter multiply'd by $\frac{1}{2}$ of the Transverse Diameter, to the Solidity of the Spheroid.

Example.

If the Conjugate Diameter of a Spheroid be 14, and the Transverse Diameter 24; What is the Solidity of the Spheroid? *Ans.* 2464. For, As 14, 11; so is 106 (the Square of the Conjugate Diameter,) multiply'd by 16 ($\frac{1}{2}$ of the Transverse Diameter), to 2464.

PROP. XV.

To find the Solidity of any of the Platonic Bodies.

The RULE.

Always extend the Compasses from 1, to the Side of the given Platonic Body; the same Extent being turn'd three times (the same way)

Solidity of the	0.11781	Tetrahedron.
	0.417404	Cube.
	2.18169	Octaedron.
	7.663119	Dodecaedron.

For, As 1 to the Cube of the Side of any of the Platonic Bodies; so are the Numbers above in this Proportion; to the Solidity of their respective Platonic Body.

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Example.

If the Side of an Octagon be 10; What is its Solidity? *Ans.* 471.404.

P R O P. XVI.

An Universal Method, to find Centres on the Line, to measure any Superficial, Solid, or Concave Measures by any kind of Measure whatsoever.

The RULE.

The Centre must always be at the Diameter of that Circle, or Side of that Square (in such Measures as the Diameter or Side is given in) that at one Measure in Length, (such as the Length is given in) may make one such Measure as the content is requir'd in, whether Applicative or Receptive. So that this Proposition is of General Use, not only in Superficial and Solid Measure, but also in Gauging, and in Statics.

This Proposition (which to my knowledge was never publish'd before,) is that, by which I have found many Hundreds of the Centres publish'd in this Book.

Thus thou hast Rules for the Solution of all Propositions in Solid Geometry: these shall suffice at present; but if this Book find Acceptance, I may hereafter (*Deo Volente*) enlarge, not only on this, but several other Particulars; and publish some other things, not commonly known to the World.

C.H.A.P.

C H A P. VI.

The Application of Superficial and Solid Measure, to the Mensuration of Artificers Work.

THE several Artificers that Work by Measure, having different Methods of Measuring their Work; I shall, in this Chapter, speak something to each of them: shewing in

S E C T. I.

The Method us'd by Carpenters in Measuring their Work.

I. In *Felling of Timber*, Carpenters commonly Work by the *Tun* or *Load*, which is a Solid Measure, containing 40 or 50 Solid Feet; viz. 40 Feet of round, and 50 of hewn Timber, is call'd a *Tun* or *Load*; which Denomination it receives from the Supposition, that 40 Feet of round, or 50 Feet hewn Timber, weighs about a *Tun* Weight, (i. e. 20 Hundred) which is commonly accounted a *Cart-Load*. The Tree being cut down, their Custom is, to gird it in the Middle of the Length, and folding the Line twice, (to take $\frac{1}{4}$ of it) they account that for the true side of the Square. And the Length they account from the Butt-End, up so far as the Tree will hold $\frac{1}{2}$ a Foot Girt, (as they phrase it,) i. e. the Line $\frac{1}{2}$ a Foot when twice folded.

The Dimensions thus taken; the Tree is measur'd by Chap. 3. Prop 1. of this Book. So, the Centre, for finding the Solidity in Feet, (the Side being given in Inches, and the Length in Feet, which is the most usual *Data*;) will be at 12. But if the Side of the Square, and the Length be given in other Measures; the Table and Rules in the foremention'd Proposition, will give the Centre.

If the Tree have any great Foughs which are Timber, as they phrase it; (i. e. which will hold $\frac{1}{2}$ a Foot girt)

girt) they commonly Measure them, and add them to the Whole. The Solidity of the Whole being thus found in Feet; divide it by 40, it brings it into Tuns.

But if you would at one Operation find the Solidity in Tuns; then, the Side being given in Inches, and the Length in Feet, (which is the most usual *Case*) the Centre will be at 74.39.

Note (1.) If round Timber be measur'd in order for Sale; they commonly (for Oak) cast away an Inch out of the Square, for the Bark; (i. e. if a Tree be 10 Inches square, they measure it as if it were but 9;) but for Ash, Elm, and Beech, an Inch is too much. (2.) That this way, of taking $\frac{1}{4}$ of the Circumference for the true Square, always gives the Solidity less than the Truth, by about a fifth part; as was hinted above, in Prop. 3. of Chap. 5.

H. In hewing of Timber, Carpenters also work by the Tun or Load; (which, as I said before, consists of round Timber 40, and of hewn Timber 50 Feet.) They having hew'd a Tree: their Custom is to find the middle between the Butt-end, and so far up as it will hold Timber, (i. e. so far as it will hold $\frac{1}{2}$ a Foot square) and there to measure the Breadth of it, by clapping a Rule (or other straight things) to the Sides of the Tree, and measuring the Distance between them; and in the same manner, they take the Breadth the other way; which if they are unequal, they add them together, and take half their Sum, which they account the true side of the Square.

Having thus taken the Dimensions; the Tree is Measured by Chap. 3. Prop. 1, so, the side of the Square being given in Inches, and the Length in Feet, (as they most commonly are) the Centre for finding the Solidity in Feet, will be at 12. But if the Dimensions are given in other Measures, the Table and Rules in the forementioned Prop. will give the Centre.

If you would at one Operation find the Solidity in Tuns; the side being given in Inches, and the Length

in Feet; (the most usual Data;) the Centre must be at 84.85.

Note: Tho' their *Methods* of taking the Dimensions of square and round Timber are both erroneous; yet Custom has made them current.

3. Carpenters Measure the Timber Frame (which some not improperly call'd the Carcass) of a Building, by the *Square*; which is a Superficial Measure of 100 Feet Square; i. e. 100 Superficial Feet. They also Measure by this Measure, the Partitioning, Timber-flooring or Joysing, Roofing, and Boarding of Floors. Therefore, the Dimensions, (*viz.* the length and breadth or height,) of any such Work being taken; it is Measur'd by Chap. 3. Prop. 1. So, the Length and Breadth being both given in Feet, (which is the most usual Data,) the Centre for finding the Area in Squares, will be at 100. But if the Dimensions are given in other Measures; the Table and Rules in the forementioned Prop. will give the Centre.

Note: (1.) In Flooring; the Well-holes for the stairs, and ways for the Chimneys, must be measured by themselves, and deducted from the whole Flooring, if the Workman find Materials; or else there need be no deduction made; unless there be an amend made in the Price.

(2.) In Roofing, there is no Deduction made for the Holes, through which the Chimney shafts pass; or the Vacancies for Northern Lights; and Sky-Lights.

S E C T. II.

The Method us'd by Brick-layers and Masons, in Measuring their Work.

Bricklayers and Masons being both the same in the Countrey, I have here joyn'd them together. Their Work is of divers kinds; 1. Walling, 2. Healing or Tying, 3. Plastering, 4. Paving, and 5. making Chimneys: Of all which, I shall speak in their Order. App.

I. of Walling.

1. Brick-layers commonly measure their Wall by the Perch or Rod, a Superficial Measure of $16\frac{1}{2}$ Feet square, or 272,25 Superficial Feet. So, if the Height and Length of a Wall be given in Feet, which is the most usual Data) the Centre for finding the Area in square Rods or Perches, will be at 272,25. But if the Dimensions be given in any other Measures; the Table and Rules in Chap. 3. Prop. 1. will find the Centre. Note: In some places, Work-men have a Custom, to allow but 16 Feet square to the Rod; and then (the Dimensions being given in Feet) the Centre for finding the Area in Square Rods, will be at 256.

2. Having thus found the Superficial Content of a Wall in Square Rods; they next consider the Thickness of their Wall; for they have a certain Standard Thickness, to which they reduce all their Walls; and this Standard is $\frac{1}{2}$ Brick thick; i. e. the Length of one Brick and the breadth of another: so that a Wall of 3 Bricks (Length) thick, of the same Height and Length with another of $1\frac{1}{2}$ Brick thick; the former will contain twice as many Square Rods as the later. Therefore, having found the Superfices of a Wall by the first Rule; say, A 3, is to the Thickness of the Wall in $\frac{1}{2}$ Bricks (i. e. in breadths of Bricks; the breadth of a Brick being always $\frac{1}{2}$ its Length) so is the Area before found, to the Area at their Standard Thickness of half a Brick.

3. Their Method of taking the Dimensions is also to be known; which is thus, for a Wall round a Garden, or the like, they commonly measure the length by a Line, going over the Buttresses; so likewise for the Height, they measure over all the Mouldings (pressing the Line into them) even to the middle of the Copping.

Note (1) If the Length of the Walls at the Ends (of a Garden, or House, be taken on the Outside (of the Garden or House,) then the Length of the Walls

Walls on the *Sides* (of the Garden or House) ought to be taken on the *Inside*. (2.) That when the Walls of a House are measured; the Doors and Windows are likewise to be measured, and deducted from the whole.

II. Of Healing or Tyling.

This is commonly measured by the Square; as Carpenters measure the Timber Frame of a House; (see Sect. 1. *Numb. 3.* of this Chap.) and there being no difficulty in taking the Dimensions; I shall not need to say any more about it: only the Reader may here

Note, That in some places, they have a Custom to be paid (oven and above the plain Work) for all *Hollow Ware*, as they call Ridge Tiles, Corner Tiles, and Gutter Tiles. But herein the Customs differ; for in some Places they account every Lineal Foot of such Work, for a Superficial Foot of Measure, and consequently, for 100 Lineal Feet of such Work, they reckon a Square of Healing. And in other places, they reckon a Superficial Foot of Measure for every ridge Tile, corner Tile or Gutter Tile; so that for a hundred such Tiles, they reckon a Square of Healing. The Reason that they urge for these Customs, is a greater Trouble of laying those Tiles, than of plain Tiles. But in these Cases, they take the Dimensions of their plain Work no farther than just to the Hollow Ware; or at most but a little way upon them, so far as they think the plain Tiles go.

III. Of Plastering.

This Work is of two Kinds, viz. (1) Work Lathed and Plastered; which some call *Ceiling*; (2) Plastering upon Brick Work, or between the Quarters of Partitioning; and these some call *Rendring*. All which are measured by the Yard Square, which is 9 Superficial Feet. There is no difficulty in taking the Dimensions; which being taken in Feet, (as they most commonly are) the Centre for finding the Area in Yards, will be at 9. If the Dimensions are given in any other Measures,

asures,

tures, the Centre may be found, by the Table and Rules in Chap. 3. Prop. 1.

Note: That in Partitioning, if the Workman find Materials; the Doors and Windows are measured by themselves, and deducted from the whole, as is also a sixth part (of the rest) for the Quarters in the Rendering Work. But if the Workman do not find Materials, there is no Allowance to be made for them; the Trouble in cutting and fitting their Laths, being equivalent to the void Spaces left for the Doors and Windows. Neither (in case of Workmanship only,) is there to be any Allowance made, (in Rendering,) for the Quarters, Braces, or Inter-Tyes; the Work being as much as (if not more than) if it were all plain.

IV. *Of Paving.*

There is no difficulty in taking the Dimensions of Paving. It is measured differently according to the Custom of the place; in some places 'tis measured by the Square, like Healing, (*V. Numb. 2. of this Section.*) And in some places 'tis measured by the Foot; and then the Dimensions being given in Feet (as they mostly are) the Centre will be at one. But in most places 'tis measured by the Yard, as Plastering; which see in Number 3. of this Section.

V. *Of making of Chimneys.*

Bricklayers commonly agree by the Hearth, for making of Chimneys; and then they are nothing to our purpose: But they also very often work by the Rod, as about other Work; and then their Method for taking the Dimensions is thus: If the Chimney stand singly and alone, not leaning against, nor being in a Wall, and it be wrought upright over the Mantle-Tree to the next Floor; Give it about the Brest for the Length, and take the Height of the Story for the Breadth, and the Thickness of the Jaws for the Thickness: But if the Chimney stand against (or in) a Wall, which is

M

before

before measur'd with the rest of the Building; then the Breadth of the Breast or Front, and the Depth of the two Jaums, is the Length; and the Height of the Story the Breadth; and the Thickness of the Jaums the Thickness. Having taken the Dimensions, measure it as Walling. V. Numb. 1. of this Section. [Note. Here is nothing to be deducted for the Vacancy between the Hearth and the Mantle-Tree; because of the Wyths, and the Thickning for the next Hearth above.] Then for the Shaft of the Chimney, Girt it about in the smallest Part, for the Breadth; and take the Length of the Shaft for the Length; which measure at a Brick thick, in consideration of the Wyths, Pargeting, and Scaffolding.

S E C T. III.

The Method us'd by Joiners, in Measuring their Work.

Joiners Measure their Work by the Yard Square, (i. e. 9 Superficial Feet.) The chief (if not the only) Work which they do by Measure, is Wainscoting, their custom is, to take the Dimensions with 2 Strings; pressing it into the Mouldings; for they say, (and it is but Reason) we ought to be paid for all where the Plain goes.

Therefore, when they would take the Dimensions of a Room they have Wainscoted, they tack up a Line on the Top of one corner of the Room; and as they carry it down to the Bottom, they press it (with their Fingers) into all the Mouldings; this they account the breadth; and they measure the Circumference of the Room for the length. Some Joiners will measure that also with a string; but some do not. The Dimensions being thus taken in Feet, (as they most commonly are) their work is measur'd by Chap. 3. Prop. 12. So the Centre, for finding the Area in Yards, will be at p. 20. But if the Dimensions are given in any other Measures, the Table and Rules in the foremention'd Prop. will give the Centre.

Note

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Note (1.) That you must make Deduction for all Window-lights, and measure the Window-boards, Cheeks, and Saphetaes by themselves.

(2) That for Window-Shutters, Cubboard-Doors, and such like things as are wrought on both Sides, they reckon Work and half; for indeed the Work is $\frac{1}{2}$ more.

(3) That Cornices, Bases, and Sub-Bases, are sometimes measur'd by the Foot lineal Measure. So also are Friezes, Architraves, and Chimney-Pieces measur'd; unless agreed for by the Great.

S E C T. IV.

The Method us'd by Painters, in Measuring their Work.

Painters measure their Work by the Yard, like the Joyners, (V. S. Sect. 3.) and their Method of taking the Dimensions of their Work, is also exactly the same; and they give the same Reason for it; (for, say they, we ought to be paid for all where the Brush goes;) And therefore I shall not need to say more about it. Only

Note (1.) That Painters never reckon Work and half; but once, twice, or thrice, &c. as done over.

(2.) That they do also very often Paint solid Bodies; but I refer you for the Mensuration of them to Chap. 4.

S E C T. V.

The Method us'd by Sawyers in Measuring their Work.

They Measure their Work by the Foot superficial. There is no difficulty in taking the Dimensions; for they reckon the Depth of the Carf, for the Breadth; and the Length, for the Length. The Dimensions being taken, viz. the Breadth in Inches, and the Length in Feet, (as most commonly they are,) their Work is measur'd by Chap. 3. Prop. 1. So the Centre (in this case) will be at 12. But if the Dimensions are given in any other Measures, (which they seldom are,) the

Table and Rules in the foremention'd Proposition will give the Centre.

Having thus cast up the Number of Feet in one Carf; say, As 15 is to the Number of Carfs of the same Depth and Length; so is the Area of one Carf measur'd, to the Area of them all. Having thus cast up their whole Work, in Feet; they are paid for it by the Hundred, for 100 Feet. Then, to find the Price of any Number of Feet under 100; say, As 100, is to the Price of 100; which in the Country is commonly (for Oak) from 32 d. to 36 d.; so is the given Number of Feet, to their Price.

Note (1.) If the Carf be but 6 Inches (or, be less than 6 Inches) in Depth; they have a Custom to be paid for *Carf and half*, (as they phrase it,) that is, for half so much more as it comes to by Measure. The Reason they urge for this Custom, is, their Trouble in Linding and Removing their Timbers.

(2.) For *Breaking Work*, (i. e. cutting a Log through the Middle,) and *Slabing*, (i. e. cutting off the Outside Pieces,) if the Carf be more than 12 or 13 Inches deep; they are paid by the Foot Lineal Measure, at 1 d. 1½ d. 2 d. 2½ d. or 3 d. See the Foot.

S E C T. VI.

The Method us'd by Glaziers, in Measuring their Work.

Glaziers measure their Work by the Superficial Foot, as the Sawyers; (V. S. Sect. 4.) There is no Difficulty in taking the Dimensions; only it is to be noted, that they are best taken in Inches, both Length and Breadth, (because they commonly take their Dimensions to ¼ of an Inch,) and then the Centre for Measuring their Work, will be at 144. If the Dimensions are given in any other Measures, the Table and Rules in Chap. 3. Prop. 1. will give the Centre.

Note (1.) That Windows with Half Rounds at the Top, are measur'd at their full Height, as if they were square. Also, Oval, or Round Windows, are measur'd

at the full Lengths and Breadths of their Diameters, as if they were square. So also, Crocket Windows in Stone-Work are measur'd by their full Squares. And the Reason they urge for all this, is, The Trouble in taking the Dimensions, the Wast of Glass in working, and the Trouble in setting it up, is (they say) more than the Glass is worth.

(2.) That Glaziers cut two Kinds of Diamond Quarries; one more acute than the other; the former, they call Long Quarries; and the later, Square Quarries. The acute Angle of the Square Quarries, is always 77 Degrees and 19 Minutes; and of the Long Quarries, 67 Degrees and 22 Minutes. Each of these are cut to different sizes, which are denominated from their Number in a Foot, as 8s. 10s. 12s. &c. that is, 8 Quarries of the Size call'd 8s. make a superficial Foot; and 10 Quarries of 10s. make a Foot; and 12 of 12s. and so of the rest.

(3.) That a Perpendicular let fall, from one of the Obtuse Angles, to the Opposite Side, is call'd the Range of a Quarry; and the longest Diagonal, is call'd the Length; and the shorter one, the Breadth: the Side and Area of a Quarry need no Explanations.

This being premis'd, I shall here add 5 Propositions, very useful for Glaziers; but before I come to their Solution, it will be necessary to Note,

(4.) That the Dimensions, and Area of a 12s. are as in the following

T A B L E.

Square Quarries.		Long Quarries.	
Dimensions.	Inches.	Dimensions.	Inches.
Range—	3. 43	Range—	3. 34
Side—	3. 41	Side—	3. 61
Breadth—	4. 38	Breadth—	4. 00
Length—	5. 47	Length—	6. 00
Area—	1. 00	Area—	1. 00

That by Inches in the Area of a Quarry, are meant Long Inches, (such as the Glaziers account by) viz. such, whereof 12 make a superficial Foot; each (being a Foot long, and an Inch broad) containing 12 superficial Inches.

PROP. I.

To find any of the Dimensions of a Quarry of Glass, of any given Size; as suppose 8 s. or 10 s. or, &c.

THE RULE.

Divide the Distance (on the Line) between 12 and the given Size, into two equal Parts; that Extent being laid the contrary way, from any of the Dimensions of a 12 s. (exhibited in the Table above in Note 4.) will reach to the respective Dimension of the given Size.

Example $\left\{ \begin{array}{l} 12 \\ 2. \end{array} \right.$

What is the Side of a $\left\{ \begin{array}{l} \text{Square} \\ \text{Long} \end{array} \right.$ Quarry of Glass, of the Size call'd 8 s? Ans. $\left\{ \begin{array}{l} 4.30 \\ 4.41 \end{array} \right.$ Inches. For the Extent from 12, half way towards 8, will reach (the contrary way) from $\left\{ \begin{array}{l} 3.51 \\ 3.61 \end{array} \right.$ (the side of a $\left\{ \begin{array}{l} \text{Square} \\ \text{Long} \end{array} \right.$ 12 s.) to $\left\{ \begin{array}{l} 4.30 \\ 4.41 \end{array} \right.$ the Side of a $\left\{ \begin{array}{l} \text{Square} \\ \text{Long} \end{array} \right.$ 8 s. sought.

By the same Method may any of the other Dimensions (viz. the Range, Breadth, or Length) be found. So that here are 4 Propositions compriz'd in one.

PROP. II.

To find the Area of a Quarry of Glass, of any given Size.

THE RULE.

Extend the Compasses from 12, to the given Size; that Extent will reach (the contrary way) from 1, (the

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(the Area of a 12 s.) to the Area of the given Size requir'd.

Example.

What is the Area of a Quarry of Glass, of the Size, call'd 8s? Ans. 1.5 Inch.

PROP. III.

Having any one of the Dimensions, (viz. Range, Side Breadth, or Length,) of a Quarry of Glass; To find the Denomination of its Size.

The R U L E.

Extend the Compasses from the respective Dimension of a 12's. (exhibited in the Table above in Note 4.) to the Dimension of your given Quarry: that Extent being turn'd twice (the contrary way) from 12, will reach to the Denomination of the Size requir'd.

Example { 1.
2.

Of what Size is that { Square } Quarry of Glass,
 whose Side is { 4.30 } Inches? Ans. 8 s. For the Ex-
 tent from { 3.51 } (the Side of a { Square } 12 s.)
 to { 4.30 } the Side of the given { Square } Quarry,
 will reach (being twice turn'd.) from 12 to 8.

By the same Method, may the same thing be found, from any of the other Dimensions; viz. Range, Breadth, or Length: So that here also are 4 Propositions compriz'd in one.

P. R. O. P.

P R O P. IV.

Having the Area of a Quarry of Glass; To find the Denomination of its Size.

The R U L E.

Extend the Compasses from 1, (the Area of a 12 s.) to the Area of the given Quarry; that Extent shall reach (the contrary way) from 12; to the Denomination of the Size requir'd.

Example.

Of what Size is that $\left\{ \begin{array}{l} \text{Square} \\ \text{Long} \end{array} \right\}$ Quarry of Glass, whose Area is 1; 5 Inch? Ans. 8 s.

P R O P. V.

To find whether a Window be Glazed with Square, or Long Quarries.

The R U L E.

Having measur'd the Length and Breadth of one of the Quarries; Extend the Compasses from the Breadth to the Length; and if that Extent being apply'd (the same way) from 4, do reach to $\left\{ \begin{array}{l} 5, \\ 6, \end{array} \right\}$ then are they

$\left\{ \begin{array}{l} \text{Square} \\ \text{Long} \end{array} \right\}$ Quarries. This needs no Example.

S E C T. VII.

The Method us'd by Surveyors in Measuring their Work.

The Work of Surveyors is Measuring and Mapping of Land; which they generally do by the Acre; a superficial Measure, containing 160 square Perches.

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I shall not here insist upon the many different Ways, which (by different Instruments) Land may be measur'd; but shall suppose the Dimensions taken; and then let the Figure of the Field be a Circle, a Quadrangle, or what else soever; it may be measur'd by Chap. 3. of this Book. If the Figure of the Field be reduc'd to a Square, or Oblong, (as the Notes in Chap. 3. Prop. 1. suppose) and the Length and Breadth be given (as they often are) in Perches; the Centre (to find the Area in Acres) will be at 160. If in Gunter's Chains, at 10.

But if the Dimensions be given (as most usually they are) in Gunter's Chains and Links, (i. e. Centesimal Parts of a Chain.) The best and readiest Method to find the Content, is, to make the Centre at 625, (the square Links in a Perch.) And saying, As 625, is to one side of a Parallelogram, in Chains and Links; so is the other side, in Chains and Links, to the Content in Perches: Which are readily reduc'd into Acres, Roods, and Perches, by the following Table.

Note: If the Figure to be measur'd, be either a Trapezia, or a Triangle; (as for the most part it will be one of them) Then, for the Trapezia, you must take but $\frac{1}{2}$ the Diagonal, and the whole Sum of the Perpendiculars; or $\frac{1}{2}$ the later, and the whole former: And for the Triangle, take $\frac{1}{2}$ the Base, and the whole Perpendicular; or $\frac{1}{2}$ the Perpendicular, and the whole Base: and with these Numbers work as if it were a Parallelogram.

By this Method, (if the Figure to be measur'd be not too large) you may work almost as nicely as (and far more expeditiously than) by the Pen. I have by this Method measur'd a Figure of 8 Acres in Content, to the nicety of $\frac{1}{4}$ a Perch.

A TABLE to reduce Square Perches into Acres, Roods, and Perches.

Perches.	Acres.	Roods.	Perches.	Acres.	Roods.
40	0	1	1000	6	1
80	0	2	1040	6	2
120	0	3	1080	6	3
160	1	0	1120	7	0
200	1	1	1160	7	1
240	1	2	1200	7	2
280	1	3	1240	7	3
320	2	0	1280	8	0
360	2	1	1320	8	1
400	2	2	1360	8	2
440	2	3	1400	8	3
480	3	0	1440	9	0
520	3	1	1480	9	1
560	3	2	1520	9	2
600	3	3	1560	9	3
640	4	0	1600	10	0
680	4	1	1640	10	1
720	4	2	1680	10	2
760	4	3	1720	10	3
800	5	0	1760	11	0
840	5	1	1800	11	1
880	5	2	1840	11	2
920	5	3	1880	11	3
960	6	0	1920	12	0

The Use of the foregoing Table.

Find, in the first or fourth Column, the given Number of Perches; and against it, in the two following Columns, you have their Value in Acres and Roods. But if you cannot find the exact Number of Perches in the Table; then find the next lesser Number; which subtract

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Subtract from the given Number, and the Remainder shews the odd Perches, over and above the Acres and Roods exhibited in the Table. Thus, e. g. 360 Perches will be found to be 2 Acres, 1 Rood. And thus 460 Perches, will be found to be 2 Acres, 3 Roods, and 20 Perches.

Note: (1.) That if the Surveyor measure upon the Landlord's Account, in order to make a Map of it; he then takes the Dimensions to the middle of the Hedges. But

(2.) If he measures for the Farmer, or Husbandman; then he takes his Dimensions no farther than the Plow, Sickle, or Sythe, may go.

(3.) That tho' by the Laws of *England*, there should be but one kind of Perch, (for measuring of Land) call'd the Statute Perch; yet there are several other Perches (call'd Customary Perches) made use of, in different Parts of Her Majesties Dominions; the Lengths of which (with the Statute Perch) are as in the following.

T A B L E.

N.	Perches Names.	Length in Feet.
1	Tenant-right, or curt Measure	12.
2	Statute Measure	16 $\frac{1}{2}$
3	Lancashire, or Irish Measure	21.
4	Cheshire Measure	24.
5	Scotch Measure	18 $\frac{1}{2}$
6	Other Customary Perches, used in different Places of this Kingdom.	18.
7		20.
8		22 $\frac{1}{2}$
9		28.

I shall here add two Propositions very necessary for Surveyors.

P R O P. I.

*Having the Area of a Field measured by one kind of Perch;
To find Area thereof, measured by another kind of Perch.*

The R U L E.

Extend the Compasses, from the Length of that Perch by which the Area was found, to the length of that Perch by which the Area is sought; that Extent being turn'd twice (the contrary way) from the given Area, will reach to the Area sought.

Example 1.

If the Area of a Field, measured by the Statute Perch, be 10 Acres; what would the Area of the same Field be, if it were measured by the Tenant-right Measure? Ans. 18.906 Acres. For the Extent from 16.5 to 12. (which Numbers are found, in the Table above, in Note 4. of this Sect.) will reach (being twice turn'd) from 10 to 18.906.

Thus, by the same Method, you may solve 12, different Propositions, from the Table above, in Note 4. of this Section.

P R O P. II.

Having the Plot of a Field, and its Area; to find by what Scale it was Plotted; i. e. by what Parts in an Inch.

First, by any Scale, measure the Area of the Plot; then say, as the Area thus found, is to the true Area; so is the Square of the Scale you try'd by, to the Square of the Scale it was Plotted by.

Therefore,

Divide the Distance between the Area found by Tryal, and the true Area, into 2 equal Parts; then that extent shall reach (the same way) from the Scale you try'd by, to the Scale it was plotted by.

Example.

Example.

Suppose the Area of Field, (as it is set down in the Plot,) be 4.5 Acres; which Plot being measured by a Scale of 12 in an Inch, the Area is found to be 2.52 Acres; what Scale was the Field plotted by? Ans. By a Scale of 16 in an Inch.

SECT. VIII.

The Method used by Pioneers, in measuring their Work.

1. Pioneers, in digging of Vaults, Cellars, and the like, commonly work by the Yard Solid, viz. 27 Solid Feet. The Dimensions being given, their Work is measured as a Parallelopipedon, by Chap. 5. Prop. 1. only Note, That depth in this Case, answers to thickness in that. So that if the breadth, depth and length, be all given in Yards; the Centre will be at 1. And if the breadth and depth be given in Feet, and the length in Yards; the Centre will be at 3. And so the Dimensions being given in other Measures; the Centre will be found by the Table and Rules before mention'd.
2. In making Ramparts, Parapets, Moats, &c. (for the Fortification of Towns;) also, in making new Rivers, Banks to keep out the Sea or Rivers, &c. they commonly work by the Floor, a solid Measure containing 324 solid Feet; which is 12 Cart Loads; for a Cart Load is, or ought to be a solid Yard, or 27 solid Feet; The Cart ought to be 2 Foot 8½ Inches broad at the Axle-tree within, 2 Foot high, and 5 Foot long. And then the breadth and depth being given in Feet, and the length in Yards; the Centre, for finding the solidity in Floors, will be at 10.392. But if the Dimensions be all given in Yards, the Centre will be at 3.464.

These are all the Artificers I can think of at present; that work by Measure, either by Superficial or Solid: There are others that work by Lineal Measure; but there being no difficulty in measuring their Work; I shall not need to say any thing of it.

N

CHAP.

C H A P. VII.

Statics, by the Line.

STATICS is the Art of finding the Gravity or Weight of any body; and (according to the common acceptation of the Word) is properly no part of Geometry; it being usually done by the Ballance: But the Gravity or Weight of a Body, may also be found from its Dimensions; and so it is properly a Part of Geometry; and in this sense I shall speak of it. And because some Authors under Statics, comprehend the finding the Value of Body's; I shall do so too; and shall (therefore) distinguish this Chapter into 2 Sections, viz. (1.) *Barytometry*; and (2.) *Timometry*.

S E C T. I.

Of Barytometry, or the finding the Gravity or Weight of any Body.

P R O P. I.

To find the Gravity or Weight, of any Quantity, of any of the Solid or Liquid Bodies, mention'd in the following Table.

It would be too tedious to lay down particular Analogies, for every one of these things; I shall therefore (here) exhibit a brief Table of Centres or Points on the Line; by which, and the following Rules, the Gravity or Weight of any Body, may be obtained, with great Ease and Exactness.

The

The TABLE.

Names of the Solids and Liquids.	Solid Inches in 1 Pound Averd.	Centre for	
		Square is at	Round is
Gold.	1.47212	1.213	1.368
Quick-silver.	1.84018	1.356	1.530
Lead.	2.36943	1.539	1.736
Silver.	2.65413	1.629	1.837
Copper.	3.03329	1.741	1.964
Hammer'd Iron. }	3.41056	1.846	2.083
Cast Iron.	3.67920	1.938	2.163
Tin.	3.68036	1.919	2.164
Marble.	9.14602	3.024	3.421
Common-tone. }	13.29257	3.645	4.113
Honey.	18.40213	4.290	4.840
Salt-Water.	25.27128	5.020	5.673
Cheese.	27.04000	5.200	5.866
Fresh-Water, or Wine. }	27.66378	5.260	5.933
Green Oak.	29.53083	5.434	6.130
Oyl.	30.66839	5.537	6.247
Dry Oak.	35.85891	5.988	6.755
Wheat.	38.63385	6.215	6.973

The use of the Preceding Table.

The Table is too plain to need any Description: And as to its use, observe the following Rules.

(1.) If the Body whose Gravity is requir'd, be either a Solid, in the form of a Cylinder; or a Liquid in such a figur'd Vessel; then, in the first Column of the Table, find the Name of the Body whose weight is requir'd, and against it in the 4th Column is the Centre: which being thus found, this is the

N. 2.

General

General Rule.

Extend the Compasses from the Centre, to the Diameter of the Cylinder in Inches; that extent being turn'd twice (the same way) from the Length in Inches, will reach to the Weight in Pounds Averdupois.

Example.

Let it be requir'd, to find the Weight of a Cheese, whose Diameter is 20 Inches, and thickness 4 Inches. Here the Centre will be found to be at 5.866, and from thence the weight of the Cheese will be found to be 46.496 Pounds, *i. e.* 5 Nails, 6 Pounds and about a half.

(2.) But if the Body to be weighed, be Elliptico-Cylindrical, (*i. e.* if the Base of the Cylinder be an Ellipsis,) then reduce the Elliptical Base into a Circle, (by Prop. 1. Rule 4. of Chap. 5. of this Book,) and then proceed as before, otherwise work by the Second Part of the Rule but now Quoted; only for Solidity in *that Case*, read Gravity (in Pounds Averdupois) in *this*; observing also, that the Dimensions be given (all) in Inches. This needs no Example.

(3.) If the Body whose weight is sought, be either a Solid in the Form of a Square Prism; or a Fluid contain'd in such a Figur'd Vessel; then in the first Column of the Table, find the Name of the Body, whose weight is requir'd, and against it in the third Column, is the Centre; which being found, this is the General

R U L E.

Extend the Compasses from the Centre to the side (in Inches) of the Base of the Square Prism; that Extent being turn'd twice (the same way) from the length (in Inches) will reach to the Gravity or Weight (in Pounds Averd.) requir'd.

Example.

Example.

If a piece of Green Oak, be 10 Inches Square, and 10 Feet (*i. e.* 120 Inches) long; what is the Gravity or Weight thereof? Ans. 406.38 Pounds Averd. For (by observing the Rule) the *Centre* for Square Green Oak, is found to be at 5.434, and from thence, the weight of the piece propos'd in this Example, is found to be 406.38 Pounds Averd.

(4.) But if the Body to be weigh'd, be Oblongo-Prismical, [*i. e.* If the Base of the Prism be an Oblong,] then, Reduce the Oblong Base into a Geometrical Square, (by Chap. 5. Prop. 1. Rule 2.) and then proceed as in Rule 3. of this Proposition. Otherwise, Work by the second Part of Rule 2. of Prop. 1. of Chap. 5. only for Solidity *there*, read Gravity *here*.

(5.) If the Body whose Weight is requir'd be Polygonio-Prismical, [I mean if the Base of the Prism be a Regular Polygon,] then reduce the Polygonial Base, into a Geometrical Square (by Chap. 1. Prop. 3.) and then proceed by Rule 3 of this Proposition.

(6.) If the Body (whose Gravity is sought) be of any other Figure (regular or irregular;) then, find the Solidity thereof in Inches, by Chap. 5. and then (by the Table in this Prop.) say, as the Number (in the second Column) against the Name of the Body, is to 1; so is the Solidity of the Body in Inches, to its Gravity or Weight in Pounds Averd.

Example.

How much will a solid Foot (*i. e.* 1728 Solid Inches) of Green Oak, Weigh? Answer, 18.51 Pounds Averd.

Thus (kind Reader) thou hast 108 Propositions comprized in this one, viz. 18. for finding the

Gravity of	{	Cylindrical	}	Gold, Quick-sil-
		Elliptic-Cylindrical		
		Prismatical		
		Oblongo-Prismatical		
		Poligonio-Prismatical		
		Other Figur'd		

ver, Lead, Silver, Copper, Hammer'd Iron, &c.

P R O P. II.

To find any of the *Questia* (of the preceding Proposition) in Pounds Troy.

The R U L E.

Find (by the preceding Proposition) the Gravity in Pounds Averd. which reduce into Pounds Troy, by Prop. 9. of this Chapter.

Otherwise,

Work by the Rules of the preceding Proposition, only make use of the Table in this.

So here (also) are 108 Propositions, and all resolvable from the Table in this (and the Rules in the preceding) Proposition. I have so largely handled the preceding Proposition, that I need not add any more in this, but only the

T A B L E.

T A B L E.

Names of the Solids and Fluids.	Cubick In- ches in One Pound Troy.	Centre for	
		Square is at	Round is at
Gold.	1.20996	1.100	1.240
Quick-silver.	1.51248	1.230	1.387
Lead.	1.94748	1.395	1.574
Silver.	2.18148	1.476	1.666
Copper.	2.49312	1.578	1.781
Hammer'd Iron.	2.80320	1.674	1.889
Cast Iron.	3.02400	1.738	1.962
Tin.	3.02496	1.739	1.963
Marble.	7.51728	2.741	3.093
Common Stone.	10.92540	3.304	3.727
Honey.	15.12504	3.889	4.387
Salt-Water.	20.77092	4.557	5.141
Cheese.	22.22456	4.714	5.318
Fresh-Water, } or Wine. }	22.73736	4.768	5.379
Green Oak.	24.27192	4.926	5.558
Oyl.	25.20828	5.020	5.664
Dry Oak.	29.47308	5.429	6.124
Wheat.	31.89084	5.647	6.371

P R O P. III.

Having any two Bodies (nam'd in the Table) both of the same Magnitude, together with the Weight of one of them; To find the Weight of the other.

This Proposition may be solved from either of the preceding Tables in this Chapter. This being premis'd, this is

The

The RULE.

As the Number (in the second Column of the Table), against the Body whose Weight is sought, is to the Number against the Body whose Weight is known; so is the Weight known, to the Weight sought.

Example.

If there be a Mass of Gold weighing 10 lb. How much will a Mass of Silver (of the same Magnitude) weigh? Ans. 5.54 lb. Averd. or 5.05 lb. Troy. For, As 2.65413, to 1.47212; so is 10 lb. (if the Weight of the Gold were given in Averd. Weight), to 5.54 lb. the Weight of a like Mass of Silver, in the same Weight, And, As 2.18148, to 1.20996; so is 10 lb. (if the Weight of the Gold were Troy Weight, to 5.05 lb. the Weight of the equal Mass of Silver, in Troy Weight.

This is plain, and needs no more Examples; and by the same Method may 306 Propositions be solved, from each of the preceding Tables in this Chapter; viz. 17 for each Body nam'd in the Table; e. g. By knowing the Weight of any Mass of Gold, or Quick-silver, or Lead, or, &c.) To find the Weight of a like and equal Mass of any of the other Bodies. So that this Proposition contains in the whole, 612 different Propositions.

P R O P. IV.

Having the Magnitude in Inches) of any Body, (mention'd in the following Table;) To find the Weight thereof (in Pounds Averd.) in Air, Oyl, Fresh-Water, Salt-Water, Honey, or Quick-silver.

The Proposition may be solved from either of the preceding Tables in this Chapter.

The TABLE.

The Names of the Bodies.	Weight of a Solid Inch (in lbs. Averd.) when Weigh'd in		
	Air.	Oyl.	Fresh-Water.
Gold.	.67929	.64669	.64315
Quick-silver,	.54342	.51082	.50728
Lead.	.42204	.38944	.38590
Silver.	.37677	.34417	.34063
Copper.	.32967	.29707	.29353
Hammer'd-Iron.	.29320	.26060	.25706
Cast-Iron.	.27182	.23922	.23568
Tin.	.27171	.23911	.23557
Marble.	.10933	.07673	.07319
Common Stone.	.07523	.04263	.03909
Honey.	.05434	.02174	.01820
Salt-Water.	.03957	.00697	.00343
Cheese.	.03698	.00438	.00084
Fresh-Water, } of Wine. }	.03614	.00354	.00000
Green Oak.	.03386	.00126	.00228
Oyl.	.03260	.00000	.00354
Dry Oak.	.02788	.00472	.00826
Wheat.	.02588	.00672	.01026

A Continuation of the preceding Table.

T A.

T A B L E.

The Names of the Bodies.	Weight of a Solid Inch (in Pounds Averd.) when Weigh'd in		
	Salt-Water.	Honey.	Quick-silver
Gold.	.63972	.62495	.13587
Quick-silver.	.50385	.48908	.00000
Lead.	.38247	.36770	.12138
Silver.	.33720	.32243	.16665
Copper.	.29010	.27533	.21375
Hammer'd Iron.	.25363	.23886	.25022
Cast Iron.	.23225	.21748	.29160
Tin.	.23214	.21737	.27171
Marble.	.06976	.05499	.43409
Common Stone.	.03566	.02089	.46819
Honey.	.01477	.00000	.48908
Salt-Water.	.00000	.01477	.50385
Cheese.	.00259	.01736	.30644
Fresh Water, or Wine.	.00343	.01820	.50728
Green Oak.	.00571	.02048	.50956
Oyl.	.00697	.02174	.51082
Dry Oak.	.01169	.02646	.51554
Wheat.	.01369	.02846	.51754

Before I lay down the Rule for Solving this Proposition, it will be necessary for the Reader to

Note: (1.) That all Bodies being weighed in a Fluid, (as Oyl, or Water, or, &c.) are lighter (than when weighed in the Air) by a Mass of that Fluid equal in Magnitude to the given Body. *e. g.* a solid Inch of Lead (which as the preceding Table shews, weighs in the Air .42204 of a Pound) when weigh'd in fresh-Water, will be lighter, by the weight of a Solid Inch of the Fluid, (*viz.* .03614 of a Pound) so that it will weigh in fresh-Water but .38590 of a Pound.

(2.) That

(2.) That a thing is said to be weighed in Oyl, or Water, (or &c.) when the Scale containing the thing to be weighed is put into the Fluid; the other remaining in the Air: But before the thing to be weigh'd is put into the Scale, the Scales must be made an *Equilibrium*, by putting some small Weights into the Scale of the Fluid.

(3.) That (in the preceding Table) in the Columns of Oyl, fresh and salt Water, Honey and Quick-silver; the Numbers below the Row of Cyphers, shew the Negative Gravity of the Body against which it stands, e.g. In the Column of fresh Water, against dry Oak, stands .00826 of a Pound; which shews that a solid Inch of dry Oak, will weigh in fresh Water .00826 of a Pound Negative; (i. e. .00826 of a Pound less than nothing) or .00826 of a Pound being added to a solid Inch of dry Oak, will make it equally heavy with the Water; and swim level with the Surface thereof.

These things being known; the Proposition is solved by the following

R U L E.

Say, as 1 solid Inch of any Body, is to its Weight in Air, Oyl, Fresh or Salt Water, Honey, or Salt Water, (which the Table plainly shews,) so is the Magnitude (in Inches) of the given body, to its weight in the same Fluid.

Examp. $\begin{cases} 1. \\ 2. \end{cases}$

Let there be a Mass of Lead, containing in Magnitude 1 solid Foot, or 1728 solid Inches; What is the Gravity or Weight thereof in $\begin{cases} \text{Air} \\ \text{Fresh Water?} \end{cases}$ Ans. it will weigh in $\begin{cases} \text{Air } 729.28512 \\ \text{Fresh Water } 666.8352 \end{cases}$ Pounds. For,

As 1 $\begin{cases} .42204 \\ .38590 \end{cases} :: 1728 : \begin{cases} 729.28512. \\ 666.8352. \end{cases}$

Thus, by the same Method may 108 different Propositions be solved, viz. 6 for every different kind of Body

Body mention'd in the Table. And the Table and Rule are so plain, as not to need another Example. Yet, least the Reader should be at a loss, in finding the *Negative Gravity* of a Body, in a Fluid specifically heavier than it self; I shall add the following

Example 3.

Let there be a piece of dry Oak, containing in Magnitude 10 solid Feet, or 17280 solid Inches; What is the *Negative Gravity* thereof in fresh Water? or (which is the same thing) what weight will it bear before it sink? Answ. 142.7328 Pounds. For, as 1 : .00826 :: 17280 : 142.7328.

P R O P. V.

Having the Weight of a Body in any one of these Fluids, viz. Air, Oyl, fresh Water, salt Water, Honey, or Quick-silver; to find its weight in any of the other.

The R U L E.

As the weight of a solid Inch, (of the given Body) in that Fluid in which the weight is given, is to the weight thereof, in that Fluid in which the weight is requir'd, (which weights of a solid Inch are easily found in the Table in the preceding Prop.) so is the weight of the Body given, to its weight requir'd.

Example 2.

If there be a Mass of Lead which weighs in the Air 100 Pound: what will the same Mass weigh in Fresh Water? Here, (from the Rule in this, and the Table in the Preceding Prop.) I say, as .42204 : .38590 :: 100 : 91.436 the Answer requir'd.

Thus by the same Method, and Table, may 540. different Propositions be solved, viz. 30 for every of the different Bodies mention'd in the Table, e, g. The weight

weight of a Body may be given in

{ Air
Oyl
Fresh Water
Salt Water
Honey
Quick-silver

and

requir'd in any other of these Fluids.

PROP. VI.

To find the Weight (in Oyl, Fresh Water, Salt Water, or Honey,) of any quantity of Metal compounded (to any known Proportion) of any 2 of the Metals mentioned in the preceding Table.

The RULE.

Say, as the weight (express'd in the Table in Prop. 4.) of a solid Inch (of one of the Metals) in Air, is to the weight of a solid Inch (of the same Metal) in the requir'd Fluid; so is the weight (in Air) of the Quantity of that Metal in the compound Mass, to its weight in the requir'd Fluid.

By the same Method, find the weight (in the requir'd Fluid) of the other quantity of the other Metal, that composes the Compound Mass. The sum of these two Weights, is the weight (in the requir'd Fluid) of the given Mass of Compound Metal.

Example.

Suppose a Metal compounded of 11 Ounces two Penny Weight (Troy) of Silver, and 18 Penny Weight of Copper; (which is now the Standard for our Sterling Silver Money in England; for 1 Pound of Silver Money contains but 11 Ounces two Penny weight of fine Silver, the other 18 Penny weight call'd the Alloy being Copper.) How much will one Pound thereof weigh in fresh Water? Ans. 10 Ounces, 16 Penny weight, 16 Grains Troy. For according to the Rule, I say, (from the Table in Prop. 4.) As .37677, to

O

.34063

.34063, so is 5328 (the Grains in 11 Ounces two Penny weight of Silver) to 4816 Grains, the weight of the pure Silver in Water. Again, I say, as .32967, to .29353; so is 422 (the Grains of Copper in one Pound of Silver Money) to 384 Grains, the weight of the Copper in Water. Then 4816 Grains added to 384 Grains, makes 5200 Grains, equal to 10 Ounces, 16 Penny weight, 16 Grains.

By the same Method may 60 different Propositions be Solved, viz. To find the weight (in Oyl, fresh Water, salt Water, or Honey,) of any of the 15 Compound Metals, may be made out of some two of these simple Metals, viz. Gold, Silver, Lead, Copper, Iron, Tin.

P R O P. VII.

To find the Weight of any piece of Silver Money, in fresh Water.

The R U L E.

Say, as 5700 (the Grains in a Pound Troy,) is to 5200 (the weight in Grains of one Pound of Silver Money, in fresh Water;) so is the weight of any piece of Silver Money, in the Air, to its weight in fresh Water.

This Proposition may be useful to detect Counterfeit Money. But since its weight in Air, is to be the *Datum* (or thing given,) I shall here Exhibit

A Table of the weight of English Silver Coins, at 5 s. 2 d. the Ounce, according to the Statute.

Coins Names.	Weight in Troy.		
	d.	w. & Grains.	Grains alone.
Five Shilling Piece.	19	8.52	464.52
Half a Crown.	9	16.26	232.26
Shilling.	3	20.90	92.90
Six-pence.	1	22.45	46.45
Groat.	1	6.96	30.96
Three-pence.	0	23.225	23.225
Two-pence.	0	15.48	15.48

P R O P.

P R O P. VIII.

To find the Weight of any Piece of Gold Coin, in Fresh Water.

The R U L E.

Say, as 5760, (the Grains in one Pound Troy,) is to 5433 (the weight in Grains, of one Pound of Sterling Gold, in fresh Water;) so is the weight of any piece of Gold Coin in the Air, to its weight in fresh Water.

This Analogy supposes the Gold Coin to be alloy'd with Silver; but if it be alloy'd with Copper, the 2d Number in the Analogy must be but 5426.

Example.

What will a Guinea piece weigh in fresh Water, its weight in Air being 129 Grains, and alloy'd with Silver? Ans. 121.6 Grains. For, as 5760 : 5433 :: 129 : 121.6.

The weight of a Gold Coin in the Air, being the Datum in this Prop. I shall here add

A Table of the weight of some of the most usual Gold Coins, currant in England.

Coines Names.	Weight in (Troy)	
	d.w.&Gr.	Gr. alone.
Guinea.	5 9	129
Jacobus 22 s.	6 6	150
Ch. br Ja. 20 s.	5 18	138
French } Pistol.	4 8	104
Spanish }		

This Proposition will also be very useful to Detect false Counterfeit Money. For if you suspect the Goodness of any piece of Gold or Silver Money; weigh it in the Air, and (if holds weight, or if not,) (find by this

or the preceding Prop.) how much it should weigh in Water; and if it agree with the weight before found, then is the piece of Money *Good*, (i.e. true Metal as it ought to be,) otherwise not. But tho' the Money be good, yet it may be too Light; which whether it be or not, its weight in Air will shew.

But for the reader detecting of False and Counterfeit Coins, I shall here add

A Table of the Weight (in fresh Water) of all the most usual Gold and Silver Coyne, at this time currant in England.

Coines Names.		Weight in (Troy)		
		d.w.&Gr.	Gr.alone.	
Gold	Guinea,	5	1.6	121.6
	Jacobus, 22 s.	5	21.4	141.4
	Ch. or Ja. 20 s.	5	10.1	130.1
	French } Pistol.	4	2.0	98.0
	Spanish }			
Silver	Five-Shilling Piece.	17	11.2	419.3
	Half-Crown.	8	17.6	209.6
	Shilling.	3	11.8	83.8
	Six-pence.	1	17.9	41.9
	Groat.	1	3.9	27.9
	Three-pence.	0	20.9	20.9
	Two-pence.	0	13.9	13.9

PROP. IX.

To reduce Pounds { Troy } into Pounds { Averd. }
 { Averd. } { Troy }

This is a double Proposition, and may be Solved by
 by this double

Analogy.

As { 731060 } so is any Number of Pounds
 { 601072 } Troy

{ Troy. } to the Number of Pounds { Averd. } equal
 { Averd. } thereto.

Example { 1.
 2.

In { 146 } Pounds { Troy } how many Pounds
 { 120 } { Averd. }

{ Averd. } Anf. { 120.
 { Troy ? } { 146.

P R O P. X.

To reduce Ounces { Troy } into Ounces { Averd.
 { Averd. } { Troy.

This proposition is also Two-fold, and may be solved
 by this Two-fold

Analogy.

As { 72 to 80 } so is any Number of Ounces { Troy
 { 80 to 73 } { Averd.
 to the Number of Ounces { Troy } equal thereto.

Example { 1.
 2.

In { 146 } Ounces { Troy } how many Ounces
 { 160 } { Averd. }

{ Averd. } Anf. { 160.
 { Troy : } { 146.

P R O P. II.

Having the { Diameter } (in Inches) of a
 { Side }
 { Globe or Sphere } of any of the Mettals, or Stones,
 { Platonick Body, }
 mention'd in the following Table ; to find its Gravity or
 Weight.

Here being 16 propositions compriz'd in one, it would
 be too tedious to lay down all their Analogies, Separatly

A Description of the preceding Table

May seem altogether Superfluous; the Table being so plain of it self. I shall therefore only say, that the { 2d Column exhibits } the Weight of a { Globe } { Tetraedron, Octaedron, &c. } (whose { Diam. } { Side } is 1 Inch) of Gold, Quick-silver, Lead, &c.

This being known, any of the *Quæsitæ* in this Proposition, may be easily found. by the following

R U L E.

When the Weight of { a Globe } { any Platonic Body } of Gold, or, &c. is requir'd; first find (by the preceding Table) the weight of { a Globe, (whose Diameter is one Inch) } { the giv. Plat. Body, whose side is 1 In. } of the given Metal, Then, the Extent of the Compasses from one, to the { Diameter } { Side } in Inches of the Globe { Platonic Body } whose weight is requir'd, will reach being three times turn'd (the same way) from the Weight found in the Table, to the weight of the Body requir'd.

Example 1.

Suppose there is a Globe or Bullet of Lead, whose Diameter is ten Inches; What is the Weight thereof? Ans. First, I find (by the Table) the Weight of a Globe of Lead, one Inch Diameter, to be .22107 of a Pound; and from thence I find the Weight of the given Globe of Lead to be 221.07 Pounds Averdupoiz.

Example 2.

What will a Hexaedron (or Cube) of Gold Weight whose side is 12 Inches? Answ. 1173.873 Pounds Averdupoiz.

These two Examples are sufficient (I think) to explain the Rule to any ordinary Capacity. And by the same

same Method of Working, may 60 different Propositions be solved, viz. To find the Weight of any Globe or Platonic Body of Gold, Quick-silver, Lead, Silver, or, &c.

PROP. XII.

Having the { Diameter } of { a Globe }
 Side { of any Platonic Body } of
 any of the Metals or Stones mention'd in the Table in
 Prop. 2. together with its weight; to find the Weight of a like
 and equal { Globe } of any other Metal or
 Platonic Body { of any other Metal or }
 Stone mentioned in the Table.

The RULE.

Say, as the weight (in the Table) of a {Globe
Pla. Bo.} of *that* Metal whose weight is given, is to the weight (in the Table) of a {Globe
Platonic Body} of *that* Metal whose Weight is requir'd; so is the weight of the {Globe
Platonic Body} given, to the Weight of the {Globe
Platonic Body} requir'd.

Example.

If there be a Globe or Bullet of Lead, weighing 221.07 Pounds; What will a Bullet of Silver of the same Diameter weigh? Answ. 197.35 Pounds. For, As. 221.07 : 197.35 :: 221.07 : 197.35.

After the same manner, and by the same Table; may
540 different Propositions be solved, viz. 90 for each
different Figure in the Table, e. g. there may be given

the Diameter and Weight of a Globe of { Gold,
Quick-silver, }
Lead, &c. }

to find the Weight of a Globe (of the same Diameter) of any of the other Metals, or Stones mentioned in the Table;

Table; which makes 9 Propositions for each Metal and Stone in the Table; and they being 10 in Number, it makes 90 for each different Figure in the Table; and there being 6 of them, (*viz.* the Globe, Tetraedron, &c.) it makes in the Whole 540 as was said before.

Thus (kind Reader) I have given thee Rules for the Solution of 2151 Problematical Propositions in Statics. And many more Rules may be easily deduced from the Preceding Tables of this Chapter: But I had thought wholly to omit them; not only (tho chiefly) for Brevity's sake; but also, because they do not so properly belong to Statics, especially as it is here consider'd under the Name of Barytometry. But since some good Authors, have not scrupl'd to admit them into the Statick Art; I shall here give some brief Hints, at the most useful of them.

I. From the 2d Column of the Table in Prop. 1. may 18 Propositions be deduced, *viz.* To find the Magnitude (in Solid-Inches) of any Body (mention'd in the Table) whose Weight is known. And this is done by this.

RULE.

Say, as 1 Pound (of any Body) is to it's Magnitude in Solid-Inches (which the Table plainly exhibits;) so is any other Weight of the like Body, to it's Magnitude in Inches.

2. } From the { 3d } Column of the same Table,
3. } { 4th }
may { 36 } Propositions be deduc'd. *viz.* 18 for finding the { Diameter of a Cylinder } of any Body
 { Side of a Square Prism } (mention'd in the Table) whose Length and Weight is given; (and this is done by dividing the Space between the given Length and Weight, into 2 equal Parts; and applying that Distance the same way from the Centre,

Centre, it will reach to the { Diameter } sought. And
 Side
 18 for finding the Length of a { Cylinder } of any
 { Square Prism }
 Body (mention'd in the Table) whose { Diameter }
 Side
 and Weight is given. And this is done, by extend-
 ing the Compasses from the Centre to the given
 { Diameter, }
 { Side, } and turning that distance twice (the
 contrary way) from the given Weight, they will reach
 to the Length requir'd.

4. There may be alike Number of like Propositions, deduc'd from the Table in Prop. 2, when the Weight is given in Pounds Troy.

5. There may (moreover) be 305 Propositions deduc'd, from either of the Tables in the first and second Prop. viz.

By having the Magnitude and Weight of any Body, (mention'd in the Table) to find the Magnitude of any other Body, (mention'd in the Table) of the same Weight

And this is done, by saying, as the Number in the 2d. Column (of the Table) against the Body whose Magnitude is given, is to the Number against the Body, whose Magnitude is sought ; So is the Magnitude given, to the Magnitude sought.

6. There may be 108 Propositions, deduc'd from the Table in Prop. 4th. viz.

To find the Magnitude (in Inches) of any Body (mention'd in the Table) whose Weight is given, in Air, Oyl, Fresh-water, Salt-water, Honey or Quick-silver.

And this is done, by saying, as the Weight (found in the Table) of a Cube-Inch of the given Body, is to 1 ; so is the Weight of the given Body, known, to its Magnitude (in Inches) sought.

7. From the Table in Prop. 11. there may be 60 Propositions deduc'd. viz,

To

To find the $\left\{ \begin{array}{l} \text{Diameter of a Globe} \\ \text{Side of a Platonick-Body} \end{array} \right\}$ (of any of the Bodies mention'd in the Table) whose Weight is known, or propos'd.

And this is done, by Dividing the Distance between the Weight (found in the Table) of a $\left\{ \begin{array}{l} \text{Globe} \\ \text{Platonick-Body} \end{array} \right\}$ whole $\left\{ \begin{array}{l} \text{Diam.} \\ \text{Side} \end{array} \right\}$ is 1, and the Weight of the given $\left\{ \begin{array}{l} \text{Globe} \\ \text{Platonick-Body} \end{array} \right\}$ into 3 equal Parts; and applying that Extent (of the Compasses) the same way from 1, it will reach to the $\left\{ \begin{array}{l} \text{Diameter} \\ \text{Side} \end{array} \right\}$ of the $\left\{ \begin{array}{l} \text{Globe} \\ \text{Platonick-Body} \end{array} \right\}$ requir'd.

8. There may also be 540 other Propositions deduced from the Table in Prop. 11. viz.

By having the $\left\{ \begin{array}{l} \text{Diam.} \\ \text{Side} \end{array} \right\}$ of $\left\{ \begin{array}{l} \text{a Globe} \\ \text{any Platonick-Body} \end{array} \right\}$ of any of the Metals or Stones mention'd in the Table; together with its Weight; to find the $\left\{ \begin{array}{l} \text{Diameter} \\ \text{Side} \end{array} \right\}$ of any other $\left\{ \begin{array}{l} \text{Globe of the same Weight} \\ \text{like, and equally heavy Pla. Body} \end{array} \right\}$ of any of the other Metals, or Stones mention'd in the Table.

And this is done by saying, as the Weight (in the Table) of a $\left\{ \begin{array}{l} \text{Globe} \\ \text{Pla. Body} \end{array} \right\}$ of that Metal whose $\left\{ \begin{array}{l} \text{Diam.} \\ \text{Side} \end{array} \right\}$ is requir'd, is to the Weight (in the Table) of a $\left\{ \begin{array}{l} \text{Globe} \\ \text{Pla. Body} \end{array} \right\}$ of that Metal whose $\left\{ \begin{array}{l} \text{Diam.} \\ \text{Side} \end{array} \right\}$ is given; so is the $\left\{ \begin{array}{l} \text{Diameter} \\ \text{Side} \end{array} \right\}$ given, to the $\left\{ \begin{array}{l} \text{Diameter} \\ \text{Side} \end{array} \right\}$ sought.

Seeth

SECT. II.

Of Timeometry, or the Art of finding the Value or Worth of Bodies.

PROP. II.

Having the { Diameter of a Globe } of Sterling Gold,
 { Side of a Platonic Body }
 or Silver given (in Inches,) to find its Value or Worth.

The Rule.

Extend the Compasses from 1, to the { Diam. }
 { Side } (in Inches) of the given
 { Globe } { Plat. Body }, the same
 Extent being turn'd three
 times (the same way) from
 the respective Value (found
 in the Table) of a
 { Globe } { Plat. Body } whose
 { Diam. } { Side } is one Inch,
 will reach to the Value of
 the { Globe } { Plat. Body } re-
 quir'd.

Example.

What will a Hexaedron
 or Cube (whose side is two
 Inches) of Sterling Gold be
 worth? Ans. 322.8096
 Pounds. For, in the Ta-
 ble, I find the Value of a Cube (whose side is one Inch)

Of Sterling		The Value (in Pounds and Decimal parts) of a	
Silver	Gold		
21.1362	4.75419.0216	Globe	Tetrae-
.7062	.1589	whole	dron
		Diamet.	whole
		is one	side is
		Inch.	one Inch.
		Globe	Oxae-
		whole	dron
		Diamet.	whole
		is one	side is
		Inch.	one Inch.
		Globe	Hexae-
		whole	dron or
		Diamet.	Cube
		is one	side is
		Inch.	one Inch.
		Globe	Icosae-
		whole	dron,
		Diamet.	whole
		is one	side is
		Inch.	one Inch.
		Globe	Dodeca-
		whole	edron,
		Diamet.	whole
		is one	side is
		Inch.	one Inch.

of Sterling Gold to be 40.3512 Pounds; whence (working by the Rule) I find the Value of a Cube (of Gold) whose side is two Inches to be 322.8096 Pounds, that is 322 $l.$ 16 $s.$ 2 $d.$

By the same Method, and from the same Table, may 12 different Propositions be solved. This (being so plain) needs no more Examples, only I think it not amiss, to bid the Reader

Note: (1) That by *Sterling Gold and Silver*, I mean (not pure Gold and Silver, but) such Gold and Silver as our English Coins are made of, viz. such as is alloy'd with a certain Quantity of baser Metal, according to the Standard.

(2) The Standard for Sterling Gold, is in the Pound Troy, 11 Ounces of fine Gold, and 1 Ounce of Silver, or Copper, call'd the Alloy. The *Antient* Gold Coins were alloy'd with Copper, but the more *Modern* ones (as *Guinea Pieces*, &c.) with Silver; which is the Reason why *those* are of a red, and *these* of a pale Yellow. Gold thus alloy'd with Silver, is by some Authors call'd Crown Gold; and now the Value of a Guinea Piece (which weighs five Penny Weight nine Grains) is by Act of Parliament settl'd at 21 $s.$ 6 $d.$ And on this Ground, the Column for Gold, in the Preceding Table, was Calculated.

(3) The Standard for Sterling Silver is 11 Ounces 2 Penny-weight Troy of fine Silver, and 18 Penny-weight of Alloy of Copper. An Ounce of sterling Silver is now (by the Statute) valu'd at 5 $s.$ 2 $d.$ And on this Ground, the Column for Silver, in the preceding Table, was Calculated.

(4) Our Gold Coin is of equal fineness with the Spanish, French, and Flemish; but our Silver Coin has not so much Alloy in it as the French or Dutch.

(5) Our English Money (according to our most Modern Authors) was first Coin'd according to the foregoing Standard, (mention'd in Note 2 and 3) at a place call'd *Sterling*, (whence it is call'd Sterling Money,)

By *Osbert* a *Saxon* King of *Northumberland*, about 830 Years since. At which time one Ounce Troy of Silver, was divided into 20 Pieces, and so esteem'd 20 Pence, that is each piece a Penny, (hence the $\frac{1}{20}$ Part of an Ounce is call'd a Penny-weight,) and so it continu'd till King *Henry* the VIth. who valu'd the Ounce at 30 *d.* then the former Pieces or Pence went for $1\frac{1}{2}$ *d.* till King *Edward* the IVth. when the same Ounce was valu'd at 40 *d.* so that each of the aforesaid pieces went for 2 *d.* Then King *Henry* the VIIIth. valu'd the same Ounce at 45 *d.* and so it continu'd till King *Edward* the VIth. and Queen *Elizabeth*, and then the same Ounce was valu'd at 60 *d.* so that every Penny Weight was in Value 3.1 *d.*

(6) From what has been said, it appears, that of Sterling Gold.

			l.	s.	d.	q.
1	{ Pound	} is worth {	48	0	0	0
	{ Ounce		4	0	0	0
	{ Penny-weight		0	4	0	0
	{ Grain		0	0	2	0

And of the Sterling Silver,

			l.	s.	d.	q.
1	{ Pound	} is worth {	3	2	0	0
	{ Ounce		0	5	2	0
	{ Penny-weight		0	0	3	0.3
	{ Grain		0	0	0	0.5

PROP. II.

Having the Weight (in Grains) of any Piece of Sterling Gold or Silver Coin, to find its Value.

This Proposition may be solved from the Table in Note 6 of the preceding Proposition; but it may be more compendiously done (by the Line of Number.) from the following

TABLE.

TABLE.

Of Sterling	The Value of one Grain Weight in Decimals of a Pound, is	
	Gold Coin	0.0083
	Silver Coin	0.0005

The Table needs no Explanation, but plainly exhibits the Value of one Grain of Gold or Silver Coin. Therefore, to solve this Proposition, this is

The RULE.

Say, as 1 Grain, is to its Value, found in the Table; so is the given weight (in Grains) of any piece of Gold or Silver Coin, to its Value.

Example.

What is a French Pistol worth, it being a Gold Coin (of equal fineness with our Sterling Gold, and) weighing 104 Grains. Ans. 17 s. 3 d. 1 q. For, as 1, .0083, so is 104 to 1632 of a Pound, that is 17 s. 3 d. 1 q.

Note: That since in the two preceding Propositions, the Value is found in Pounds and Decimal Parts of a Pound, I shall here shew.

A ready way to find the Value of any Decimal of a Pound, in Shillings, Pence, and Farthings.

The RULE.

Double the Primes, it gives Shillings; then take the seconds and thirds together as one Number, which lessen by one, it gives so many Farthings, to be added to the Shillings before found. But if the seconds are 5 or more, Subtract 5 from them, (and for the 5 reckon one Shilling,) and reckon the Remainder with the thirds as one Number, and work as before. But if when the seconds and thirds are taken together as one Number, they exceed not 24, they are not to be lessen'd by one,

Thus you may easily find the Value of any Decimal of a Pound, only by Inspection.

Examp. $\left\{ \begin{array}{l} 1. \\ 2. \\ 3. \end{array} \right.$

What is the Value of $\left\{ \begin{array}{l} .736 \\ .884 \\ .662 \end{array} \right\}$ of a Pound ?

Answ. $\left\{ \begin{array}{l} 14s. 8d. 3q. \\ 17 \quad 8 \quad 1 \\ 13 \quad 3 \quad 0 \end{array} \right.$

Thus much shall suffice (at this time) for the Use of the Line of Numbers in the Staticks. I come now to

C H A P. VIII.

Gauging by the Line.

Gauging I define to be the Art of Measuring the Concavity of a Body: And it differs from Measuring the Solidity of a Body only in this, that the Solidity of a Body is measured by some Applicative Measure; as Feet, Yards or the like; but the Concavity of a Body is Measured by some Receptive Measure; as Pints, Gallons, Bushels, or the like.

This being Premised; I come now to shew how, in

P R O P. I.

To Gauge any Concave Vessel whatsoever, by this

General R U L E.

When any Concave Vessel is requir'd to be Gauged; consider the Figure of its Concavity; as whether it be a Cylinder, (as in a Pail or Bucket, &c.) the Frustum of a Cone, (as in Tubs, Stans, &c.) a Parallelopipidon, (as in Beds, or Coolers, Cisterns, &c.) or the like: and ha-

ving

ving taken the Dimensions in Inches; find the Concavity in Cube-Inches (by Chap. 5. of this Book) as if it had been a Solid.

Having thus found the Concavity in Cube-Inches; you may reduce it into Gallons, Barrels, or Bushels, &c. by dividing the same by the Number of Cube-Inches in each Measure, respectively: which Numbers are as in the following

T A B L E.

There is in		Cube-In	Cube-In.
A Gallon	{ Wine-Measure,	231	224
	{ Ale or Beer-Measure,	282	282
	{ Corn or Dry-Meas.	272 $\frac{1}{2}$	268.8
A Barrel of	{ Ale	9204	9204
	{ Beer	10152	10152
A Bushel, Corn-Measure,		2178	2150

Note 1. (1) There has been a great Controversy about the true Number of Cube Inches in each of these Gallons; which I shall here endeavour to decide.

(2) The *Wine-Gallon*, is by most Men thought to contain 231 Cube-Inches, yet Mr. *Mayne* believes, that if it were carefully examined, it would be found to be contain less: And Dr. *Wybard* (in his *Taſſometry*) pleads strongly, that it contains but 224 or 225 Cube-Inches at the most. This controversy gave birth to Mr. *Walker's* Experiment, (in 1688) who caus'd a Brazen Vessel to be made, (by Mr. *Isaac Carver*, an ingenious Workman,) in the form of a Parallelopipedon, whose Base was 4 Inches, and depth 14 Inches, (as Mr *Gunton* had done that for the Ale-Quart in the Exchequer;) by which it was found to contain only 224 Inches, for the Liquor in this, exactly fill'd the *Standard* in *Guild-Hall, London*, (which is unanimously agreed to be the true and Antient Measure,) in the presence of the Right Honourable Sir *John Shorter* then Lord-Mayor, Sir *Basil Fire-*

bruce Sheriff, Mr. Halley, Mr. Flamstead, and several other Learned and Worthy Gentlemen. This settl'd by Law, would improve the Crown Revenues some Thousands per Annum.

(3) The Ale-Gallon, was antiently reckon'd to contain 288½ Cube-Inches: But by the Care and Pains of Mr. Nicholas Gunton, (v. g. Note 2.) the just quantity of the Ale-Quart, (remaining in the Hands of the Chamberlaines of the Exchequer) appears to be $70\frac{1}{2}$ Cube-Inches; that is 282 in the Gallon; and so it is now agreed upon between the Collectors of the Excise and the Brewers. The Ale-Gallon in Ireland contains only 217.6 Cube-Inches.

(4) The Corn-Gallon, is by most Men, thought to contain 272½ Cube-Inches; yet Mr. Mayne believes that this Gallon (also) if it were carefully examin'd, it would be found to contain less. And the Winchester-Bushel (which, by Act of Parliament for the Malt-Tax, is esteem'd a Legal Bushel, according to the Standard in the Exchequer, is 18.5 Inches Diameter, and 8 Inches Deep; and consequently the Gallon of Corn or Dry Measure contains but 268.8 Cube-Inches; and the Bushel 215.0 such Inches.

(5) That the second Column of the preceding Table shews the Number of Cube-Inches (in each respective Measure) according to the most commonly receiv'd Opinion; and as they are us'd by most Gaugers: But the last Column, shew the true Number of Cube-Inches in each.

(6) That the Barrel of $\left\{ \begin{array}{l} \text{Ale} \\ \text{Beer} \end{array} \right\}$ contains $\left\{ \begin{array}{l} 32 \\ 36 \end{array} \right\}$ Gallons, and so the Numbers in the foregoing Table supposes it. But (by an Act of Parliament, made in the Reign of his late Majesty, King William the III. of Glorious Memory,) the Barrel of Ale and Beer (in all Places out of the Liberties of London) are (now) to be equal; and to contain each 34 Gallons; and then the Number of Cube-Inches in a Barrel will be 9598.

(7) That

(1) That though this Rule for Gauging (laid down in this Prop.) be General, and be withal very exact; yet is it also very tedious. Therefore I shall shew, in

P R O P. II.

A more speedy way, to Gauge all kinds of Vessels, and that in Wine, Ale, or Dry-Measure.

To avoid Tediumity, I shall here exhibit a brief Table of Gauge-Points or Centres on the Line, by which and the following Rules, the Concavity of any Vessel, may easily and speedily be obtain'd.

The T A B L E.

The Gauge-Point, for	The Base of the Vessel			
	Squa. is	Round is	Square is	Round is
Gallons of  Wine Ale or Beer Corn	15.199	17.15	14.967	16.884
	16.792	18.95	16.792	18.95
	16.500	18.62	16.395	18.496
	46.668	52.66	46.368	52.370
Corn-Bushel.				

1. This Method of Gauging by a Centre or Gauge-Point, always supposes the Vessel (to be Gauged) to be either a Cylinder, or a Parallelopipedon whose Base is a Geometrical Square: Therefore, when the Vessel to be Gauged is neither of these, it must be reduced to one of them.

(2) This being known, the Use of the Table is very easy; e. g. If the Concavity of any Vessel is requir'd in Wine Gallons, and the Base of the Vessel be *Square*, the Gauge-Point is at 15.199, and if the Base of the Vessel be *Round*, the Gauge-Point is at 17.15 as the Table plainly exhibits. In the same manner, the second and third Columns of the Table, shews any of the other Gauge-Points, according to the most commonly receiv'd Opinion of the Number of Cube-Inches in a Gallon. But if you would Gauge truly and exactly, (according to

to the Standard Measure;) you must find the Gauge-Points in the two last Columns of the Table; which were Calculated from the Numbers, in the last Column of the Table in Prop. 1.

3, If the Vessel to be Gauged be a *Cylinder*; (as a Pail or Bucket, or those Vessels made for Bushels, half Bushels, or Gallons, &c.) the Dimensions being taken (as they must always be) in Inches; this is

The R U L E.

Extend the Compasses from the (respective) Gauge-Point, to the Diameter in Inches; that Extent being turn'd twice (the same way) from the depth or length in Inches, will reach to the Concavity in such Measure as your Gauge-Point was for.

Example.

Let there be a Pail or Bucket, whose Diameter is 10 Inches, and depth 13 Inches, and let it be requir'd to find its Concavity in Ale-Gallons. In this Case, I extend the Compasses from 18.95 (the Gauge-Point for Ale-Gallons in a round Vessel) to 10, (the given Diameter in Inches,) and turning the same Extent twice (the same way) from 13, (the depth in Inches) it reaches to 3.64 Gallons. i. e. 3 Gallons and 5.15 Pints.

Note: The Decimal parts of a Gallon, are easily reduced into Pints, by Book 1. Chap. 4. Prop. 6.

4. If the Vessel to be Gauged be the *Frustrum of a Cone*, (such are Tubs, Stans, most Brewers Tuns, &c.) it must be reduced to a *Cylinder*; by finding a *Mean Diameter*, thus ——— Take the Diameter of the Vessel at the top, and likewise at the bottom, add them together, and take half their Sum for a *Mean Diameter*: by which Measure the Vessel by the second Rule foregoing.

I know, and confess, this Method of finding a *Mean Diameter*, is not absolutely exact; but gives the Concavity (always somewhat) too little; yet it is an Approximation

imation near enough the Truth for common Practice. For if the greater Diameter should exceed the Lesser, by $\frac{1}{4}$ of it self, (it will seldom do much more) yet this Method gives the *Concavity* false, (*viz.* too little) but the $\frac{1}{4}$ Part of it self; which is but one Pint, in 135.4 Gallons; a Matter of no Moment; for in a Vessel of that Capacity, the irregularity of its Figure, caus'd by the swelling of the Wood, will produce as great an Error as that amounts to. For in *all* (especially large) Vessels made by the Cooper; the swelling of the Staves, makes the Circumference of the Vessel, rather Polyhedral than Circular; and the bottom, by swelling is made very extuberant and uneven, and many times, somewhat Elliptical; by reason it swells more by the Breadth of the Wood, then by the Length. And seeing the Circle is of all others the most capacious Figure; it necessarily follows, that all those Irregularities, detract somewhat from the Natural Capacity of the Vessel. Therefore, though the Rule laid down (above) for finding a Mean Diameter, be not the exactest in the Theory; yet I believe it will be often found to be so in Practice.

5. If the Vessel to be Gauged be *Elliptico-prismital*, i. e. if it be an Elliptical Prism, (*viz.* the top and bottom be Ellipses of equal Dimensions;) then having taken the Transverse and Conjugate Diameters, divide the Space (on the Line) between them, into two equal Parts, and the Compasses shall rest on a Mean Diameter of a Circle, equal to the Ellipsis; by this Mean Diameter measure the Vessel, by the third Rule foregoing.

Or else Work thus:

Extend the Compasses from the respective Gauge Point to one of the Diameters of the Ellipsis, the same Extent being apply'd (the same way) from the other Diameter of the Ellipsis, will reach to a fourth. Then again, the Extent from the Gauge-Point, to this fourth will reach (the same way) from the depth, to the Concavity,

cavity, in such Measures as the Gauge-Point respects.

6. If the Vessel to be Gauged be the Frustum of an Elliptical Pyramid; (such I have seen some Keelers or Kives, and such may be some large Tuns, for the Reason delivered in the fourth Rule; Having taken the Dimensions of each Ellipsis, reduce them into Circles, as in the 5th Rule; then the Vessel is reduced into a Cylinder, by the fourth Rule (of this Prop.) and then 'tis measur'd by the third Rule.

7. If the Vessel to be Gauged be a Cask; as a Barrel, Hoghead, or the like; it must also be reduced to a Cylinder, by finding a Mean-Diameter, thus ———

Consider the Form of the Cask; and if it be a
 $\left\{ \begin{array}{l} \text{Spheroid,} \\ \text{Parabolo d,} \\ \text{Conoid,} \end{array} \right\}$ say, as $\left\{ \begin{array}{l} 1, .7 \\ 1, .63 \\ 1, .15 \end{array} \right\}$; so is the difference

of the Diameters of the Head and Bung, to a fourth, which fourth added to the *Lesser Diameter* (i. e. the Diameter of the Head) gives the *Mean Diameter*. By this *Mean Diameter*, Measure the Cask by the third Rule.

8. If the Vessel to be Gauged be a *Parallelopipedon* or *Square Prism*; (i. e. if the top and bottom be Geometrical Squares of equal Dimensions) then, to Gauge it, this is the

R U L E.

Extend the Compasses, from the respective Gauge-Point, to the side of the Square, (in Inches;) the same Extent of the Compasses, being turn'd twice (the same way) from the depth or length (in Inches,) will reach the Concavity, in such Measures as your Gauge-Point was for.

Example.

Let there be a Cooler (or the like) 5 Feet (equal to 60 Inches) Square, and 10 Inches deep; what is its Concavity in Ale-Gallons? Here I extend the Compasses from 16.702 (the Gauge-Point for Ale-Gallons

in

in a Square Vessel,) to 60 (the side of the Square in Inches) the same Extent being twice turn'd (the same way) from 10, (the Depth in Inches) reaches to 128 Gallons, the Concavity sought.

9. If the Vessel to be Gauged be a Frustum of a Square Pyramid, it must be reduced to a Square-Prism, by finding the side of a Mean square, thus ———

Measure the side of the Square at the bottom, and likewise at the top, add them together, and take half their Sum for the side of a Mean Square; by which Measure the Vessel, by the eight Rule.

I confess, this Rule, for finding the side of a mean Square, is not absolutely exact, but is subject to the same Deficiency as that for finding a mean Diameter for the Frustum of a Cone; (v. Rule 4.) viz. a mean Square thus found, gives the Concavity too little the $\frac{1}{16}$ part of it self; which is inconsiderable, and not worth the minding; especially if we consider, how much easier this Method is, than the true way of Measuring the Frustum of a Pyramid.

10. If the Vessel to be Gauged be an Oblong-Prism, [i. e. if the top and bottom be Oblongs or long Squares, of equal Dimensions] then having taken the length and breadth, in Inches, of one of the Oblongs, divide the Space on the Line betwixt them, into two equal Parts, and the Compasses will rest on the side of a mean Square, equal to the Oblong. The Vessel being thus reduc'd to a square Prism, measure it by the eight Rule foregoing.

11. But if both the top and bottom of the Vessel to be Gauged be Oblongs, and of different Dimensions, (such a Figure I may call the Frustum of an Oblong Pyramid) then reduce each Oblong into a Square, by the tenth Rule; and so the Vessel will be reduc'd to the Frustum of a Square Pyramid, and may be measured by the ninth Rule.

These are all the usual Figures of Vessels to be Gauged therefore I shall only add a few brief Rules, to be observed

serv'd in taking the Dimensions, and so conclude this Proposition. Therefore,

12. First, because the length of Casks can't easily be taken on the *Inside*, it must be done on the outside, remembering to deduct (from the Length so taken) the depth of the Chines, and the thickness of the Heads can't easily be measur'd, therefore know, that ———

The common size of the Thickness of the Heads of

- { Pipe, Punchion, is 1 Inch.
 { Hoghead, is $\frac{1}{2}$ of an Inch.
 { Barrel, and other smaller Vessels, about $\frac{1}{4}$ an Inch.

13. Secondly, in taking the Dimensions of large Tuns, &c. you ought to be very nice, and take the Diameter several ways; for tho' the Vessel were made Circular, yet they commonly become Elliptical, for the Reasons in the fourth Rule.

14. Thirdly, in Gauging the Liquor in Brewers Tuns or Coolers, observe, that 'tis Customary to set these Vessels inclining a little more towards one side than the other, for the Conveniency of taking out the Liquor; and therefore the best way in this case, is to pour in so much Water as will just cover the bottom of the Vessel, measuring it as you pour it in; or else when it is in, take half the depth of it in the deepest place for the true depth, supposing it to cover the bottom equally; and measure it by some of the preceding Rules: so having found the Quantity of Water poured in, always keep it as a Reserve; as you must also the half depth in the deepest place. But the former way of Measuring the Water poured in is the best; by reason of the Irregularity of the bottom of most large Vessels, mention'd in Rule 4.

Then, having also observ'd in pouring in the Water, which side of the Vessel it came first to, and mark'd that side of the Vessel; you must always observe for the future, to take the depth of the Liquor on that side, and from the depth so taken, subtract your reserved half Depth of the Water poured in, and by the remaining Depth,

Depth, Gauge the Liquor, by some of the foregoing Rules, of this Proposition; and to the Quantity of Liquor thus found, add your Reserve, so have you the true quantity of Liquor sought.

But if the Vessel (that stands thus inclining) be the Frustum of a Cone or Pyramid; you must know the Diameter of the Vessel, at the Surface of the Liquor, and also at the Surface of the Reserve: but these Diameters must not be taken on the Surface of the Liquor as it stands in the Inclining Vessel; for that would be too large; but they must be taken on the Surface of the Liquor supposing it to lye Parallel with the bottom; and therefore must be found by Calculation, thus

Say, as the whole Depth of the Vessel, is to the Difference of the Diameters at the top and bottom, so is the reserved Depth, to a fourth; which fourth, added to

{ Subtracted from } the Diameter of the bottom,

the Vessel be { bigger } at the top than at the bottom

gives the Diameter of the Vessel at the Surface of the Reserve, supposing it to lye Parallel with the bottom.

This Diameter you may always keep, as being of constant use, so long as the Vessel stands in the same Position; and therefore Note it in your Book, or mark it on the Vessel.

Then, to find the Diameter on the Surface of the Liquor; First Measure the Depth thereof in the deepest place, and from it subtract the reserved half Depth, and call the Remainder, the depth of the Liquor: Also Subtract the reserved half depth from the whole depth of the Vessel; and then supposing this remainder to be the whole depth of the Vessel; and the reserved Diameter to be the Diameter of the bottom; you may by the foregoing Analogy, find the Diameter of the Vessel, at the Surface of the Liquor. Then Measure the Liquor by the fourth or ninth Rule of this Proposition; adding to the quantity thus found your Reserve, so you will

Q

have

have the true Quantity of Liquor in the inclining Vessel.

P R O P. III.

To find the Quantity of Liquor in any Vessel partly full.

Respect being had the Figure of the Vessel, there is no difference betwixt this, and the preceding Proposition: only here the Depth of the Liquor is taken as the depth of the Vessel: And if the Vessel be the Frustum of a Cone, or Pyramid, then the Diameter of the Vessel at the Surface of the Liquor, is taken as the Diameter of the top. And this is all the difference betwixt Gauging a Vessel to see how much it will hold, and Gauging the Liquor in a Vessel: except the Vessel partly full be a Cask: And in that Case Work by the following

P R O P. IV.

To find the Quantity of Liquor in a Cask partly full.

To solve this Proposition without a Table is a tedious piece of Work, therefore I shall here add

A Table of Areas of Segments

D.	Area.	D.	Area.	D.	Area.	D.	Area.
1	.0017	26	.2066	51	.5127	76	.8155
2	.0048	27	.2178	52	.5225	77	.8263
3	.0087	28	.2292	53	.5382	78	.8369
4	.0134	29	.2407	54	.5509	79	.8473
5	.0187	30	.2523	55	.5636	80	.8576
6	.0245	31	.2640	56	.5762	81	.8677
7	.0308	32	.2759	57	.5882	82	.8776
8	.0375	33	.2878	58	.6014	83	.8873
9	.0446	34	.2998	59	.6140	84	.8967
10	.0520	35	.3119	60	.6265	85	.9059
11	.0598	36	.3241	61	.6389	86	.9149
12	.0680	37	.3364	62	.6513	87	.9236
13	.0764	38	.3487	63	.6636	88	.9320
14	.0851	39	.3611	64	.6759	89	.9402
15	.0941	40	.3735	65	.6881	90	.9480
16	.1033	41	.3860	66	.7002	91	.9554
17	.1127	42	.3986	67	.7122	92	.9625
18	.1224	43	.4112	68	.7241	93	.9692
19	.1323	44	.4238	69	.7360	94	.9755
20	.1424	45	.4364	70	.7477	95	.9813
21	.1527	46	.4491	71	.7593	96	.9866
22	.1631	47	.4618	72	.7708	97	.9913
23	.1737	48	.4745	73	.7822	98	.9952
24	.1845	49	.4873	74	.7934	99	.9983
25	.1955	50	.5000	75	.8045	100	1.0000

Now to find the Quantity of Liquor in a Cask, partly full, this is

The R U L E.

First, find the whole Concavity of the Cask; (by Rule 7. of Prop. 2. of this Chap.) Then, putting your Rule in at the Bung-hole, take the Diameter there, and also Note how far your Rule is wet with the Liquor,

Q 2

and

and call that the Wet Part of the Diameter; then say,

As the whole Diameter, is to the wet part thereof; so is a 100 to a 4th, which being found in the preceding Table, (under D.) you will find against it, in the next Column, (under Area,) a Decimal Number, which call a 5th; then say, as 1 is to this 5th; so is the whole Content of the Cask, to the Quantity of Liquor in it.

Example.

If the Diameter of a Cask at the Bung be 28 Inches, the wet part of the Diameter 21 Inches, and the whole Content 60 Gallons; how much Liquor is there in this Cask? *Ans.* 48.27 Gallons.

Note; This Rule supposes the Axis of the Cask to lye Horizontal; i. e. it supposes the Cask to stand in its natural and usual posture.

P R O P. V.

To Gauge the Liquor in a Brewer's Copper.

To do this, you must first Gauge the *Rising Crown*, which (being the Segment of a Globe) may be done by this

R U L E.

Pour in Water, till you have just cover'd the Rising-Crown; then take the Diameter of the Copper, at the Surface of the Water; also take the depth of the Water, in the deepest place. The Dimensions being thus taken in Inches; Measure it as the Segment of a Globe, by Chap. 4. Prop. 10 or 11 of this Book; and more expeditiously by Prop. 12 of the same Chapter. The Content being thus found in Inches; you may reduce it into Gallons, by Prop. 1 of this Chapter.

Having found the Content of the Rising-Crown; keep it as a Reserve; as you must also the depth of the Water; which depth must always be subtracted from the depth of the Liquor, and the rest of the Liquor must be

be measured as a Cylinder, or the Frustrum of a Cone, according to the Figure of the Copper; and to the Content thereof add your Reserve: So have you the whole Content of the Liquor; and if the Copper were full, of the whole Copper.

PROP. VI.

To Gauge a Sphere or hollow Globe.

This Proposition is more of curiosity than any great use; yet it may be useful to find the Concavity of Granadoes, Bombs, or Carcasses. And be solv'd by this

RULE.

If you desire to find the Concavity in $\left\{ \begin{array}{l} \text{Wine} \\ \text{Ale} \\ \text{Corn} \end{array} \right\}$ Gal-

lons; always extend the Compasses from $\left\{ \begin{array}{l} 7.69 \\ 8.13 \\ 8.04 \end{array} \right\}$ to the given Diameter in Inches, and that turn'd 3 times (the same way) from one, will reach the Concavity in

$\left\{ \begin{array}{l} \text{Wine} \\ \text{Ale} \\ \text{Corn} \end{array} \right\}$ Gallons, requir'd.

Examp. $\left\{ \begin{array}{l} 1. \\ 2. \\ 3. \end{array} \right.$

If the Diameter of a Sphere be 10 Inches; what is its Concavity in $\left\{ \begin{array}{l} \text{Wine} \\ \text{Ale} \\ \text{Corn} \end{array} \right\}$ Gallons? Answ. $\left\{ \begin{array}{l} 2.267 \\ 1.699 \\ 1.923 \end{array} \right\}$ Gallons.

PROP. VII.

To reduce any Number of Gallons of one kind of Measure, into Gallons of another kind of Measure.

To solve this Sextuple Proposition, take the three fix'd Numbers in the following

Q 3

T A.

TABLE.

Fixed Numbers for	Wine	Ale	Corn
	308	376	362

This being known; if you have any Number of Gallons of one kind, which you would reduce to Gallons of another kind of Measure, this is

THE RULE.

Say, as the Number (in the Table) under that kind of Gallon to which you would reduce your given Gallons, is to the Number (in the Table) under that kind of Gallon from which you would reduce them; so is your given Number of Gallons, to the Number of Gallons sought.

Example.

Let it be requir'd to reduce 616 Gallons of Ale Measure, into Gallons of Wine Measure. To do which, I say, as 308 : 376 :: 616 : 752, the Number of Wine Gallons, equal to 616 Ale Gallons.

I had thought here to put an end to Gauging; but in pursuance to the late Act of Parliament, I shall add

P R O P. VIII.

To Gauge a Heap of Corn.

For the more easy solving this Proposition, it will be convenient to sling up the Corn into a round or square heap, in the middle of the room; and taking it level on the top, it will be reduced to the Frustum either of a Cone, or square Pyramid, and so may be measured by the 4th or 5th Rule of the second Proposition of this Chapter.

Or you may sling the Corn up against one Corner of a Room; but that will reduce it into a Triangular Figure, and therefore more difficult to measure, than the Frustum of a Cone, or Pyramid: so I think the former way the best.

B O O K

BOOK III.

Of the Use of the Line of Numbers,
in Trigonometry.

P R O E M E.

1. **U**nder the general Title of Trigonometry, I shall in this Book (besides Trigonometry properly so call'd) comprehend (also) Astronomy, Geography, Navigation and Dialing; because what I shall here teach of these Sciences, is perform'd Trigonometrically.
2. In speaking of Trigonometry, properly so call'd, I shall first discourse of it in General; shewing how (without Tables) to find the Sine, Tangent, Secant, Chord, Versed-Line, &c. of any given Arch, or Angle, answering to any given Sine, Tangent, &c. And consequently, I shall shew how to perform the whole Doctrine of Trigonometry (as comprehending Astronomy, Geography, &c.) without Tables, by the Line of Numbers only. And this Discourse is altogether new, being wholly my own. Then (proceed in the second place) to speak more particularly of all the several cases of Plain and Spherical Trigonometry, both Right-Angl'd and Oblique.
3. In treating of Astronomy, I shall shew how to find the Rising, Southing, Setting, Declination, Longitude, Latitude, Amplitude, Altitude, Azimuth, Right and Oblique Ascension and Descension, &c. of the Planets and fixed Stars. And also, several ways of finding the time of the Day and Night by them.
4. In Geography, I shall shew how to find the Bearing

ing and Distance of any two places, whose Longitude and Latitude is known, &c.

5. In Navigation, I shall shew how to solve all the usual Propositions concerning Longitude, Latitude, Rumb, and Distance, &c. And —

6. In Dialling, I shall shew how to find all the Requisites belonging to a Sun-Dial; as the Hour Distances, stile's hight, Distance of the Substile from the Meridian, &c.

CHAP. I.

Of Trigonometry.

BEFORE I proceed to the Solution of particular Propositions, I shall explain some Symbolical Characters, which for Brevity I shall make use of, in the following Discourse: And they are these —

R. Radius.

S. Sine.

T. Tangent.

Sec. Secant.

V. S. Versed-Sine.

Co-S. Co-Sine.

Co-T. Co-Tangent.

B. Base.

P. Perpendicular.

L. (*Latus* or) Side.

ls. Sides.

Z. Sum.

X. Difference.

A. an Angle.

PROP. I.

To find the Sine of any Arch, or Angle.

To solve this Proposition, your Analogy must vary according to the Quantity of your given Arch or Angle. For, if the Quantity of your given Arch or Angle

be

1	57.1
2	57.2
3	57.3
&c.	&c.
10	58.
11	58.1
12	58.2
&c.	&c.
20	59.
21	59.1
be < 22 > Degrees, you must say, as < 59.2 >	
&c.	&c.
30	60.
31	60.1
32	60.2
&c.	&c.
40	61.
41	61.1
42	61.2
&c.	&c.

to the Quantity of the given Arch or Angle, (if less than 45 Degrees,) or to its Complement to 90 (if greater;) is Radius (which you may suppose 100, 1000 or 10000, as you please,) to the Sine of the given Arch or Angle, (if less than 45 Degrees,) or to its Co-Sine, if greater. (So if the given Arch or Angle be greater than 45 Degrees; having thus found its Co Sine, you may find its Sine by Prop 2.)

Example. If the given Arch or Angle be 30 Degrees; What is the Sine thereof? (Supposing the R. to be 1000) Answ. 500. For, as $60 : 30 :: 1000 : 500$.

Thus you see, (if you mind the preceding Table) that tho' the Analogies vary, yet they are easily remembered.

Notes. I do not pretend that this Rule is *always* absolutely exact; (tho' sometimes it be,) yet 'tis an Approximation near enough the Truth for common Practice.

I know indeed by Experience) that in Calculating Eclipses, &c. the Sines and Tangents must be found to a greater Nicety, than can (always) be done by this Rule: But I know also, that they cannot be otherwise found, without many tedious Operations in Arithmetic: And this Rule will *sometimes* give the exact Sine; and I think it will never give it false above the $\frac{1}{100}$ part of the Radius; a Matter of little or no Moment in common Practice.

P R O P. II.

To find the Sine of any Arch or Angle; Knowing the Co-Sine thereof.

The R U L E.

From the square of the R. subtract the square of the Co-S. the Remainder is the square of the Sine: Therefore, extract the square Root of the Remainder, and you have what you want.

Example.

If the S. of 30 Degrees be 500, what is the S. of 60 Degrees? Ans. 866.

P R O P. III.

To find the Co-Sine of any Arch or Angle.

The R U L E.

If the given Arch or Angle be greater than 45 Degrees, its Co-S. is readily found by Prop. 1. But if it be less; then find its S. by that Prop. Then subtract the square of the S. (thus found) from the square of the R. and the square Root of the Remainder is the Co-S. sought.

Example.

What is the Co-Sine of 60 Degrees? Answ. (Supposing the R. 1000) 500.

P R O P.

PROP. IV.

To find the Tangent of any Arch or Angle.

The RULE.

First find the Sine of the given Arch, or Angle, by Prop. 1. and the Co-Sine by Prop. 3. Then say, as the Co-S : S :: R : Tangent. This needs no Example.

PROP. V.

To find the Co-Tangent of any Arch or Angle.

The RULE.

First find the Sine of the given Arch or Angle, by Prop. 1. and the Co-Sine by Prop. 3. Then say, as the S : Co-S :: R : Co-T.

PROP. VI.

To find the Secant of any Arch, or Angle.

The RULE.

First find the Co-Sine, by Prop. 3. Then say, as the Co-S : R :: R : Secant.

PROP. VII.

To find the Co-Secant of any Arch or Angle.

The RULE.

First find the Sine by Prop. 1. Then say, as the S : R :: R : Co-Secant.

PROP. VIII.

To find the Versed-Sine of any Arch or Angle.

The RULE.

First find the Co-Sine, by Prop. 3. Then the Excess of the Radius above the Co-S is equal to the Versed-Sine.

PROP.

P R O P. IX.

To find the Co-Versed-Sine of any Arch or Angle.

The R U L E.

First find the Sine, by Prop. 1. Then the Excess of the Radius above the S. is equal to the Co-versed-Sine.

P R O P. X.

To find the Chord of any Arch or Angle.

The R U L E.

First find the Sine, by Prop. 1. and the V-S. by Prop. 9. Then the square of the S. added to the square of the V-S. is equal to the square of the Chord.

Otherwise,

Take half the given Arch or Angle, and find the S. thereof, by Prop. 1. double the Sine thus found, it gives you the Chord sought. This last is much the easiest and speediest way of solving this Proposition.

P R O P. XI.

To find the Co-Chord of any Arch or Angle.

The R U L E.

First, find the Co-S. by Prop. 3. and the Co-V-S. by Prop. 9. Then the square of the Co-S. added to the square of the Co-V-S. is equal to the square of the Co-Chord.

Otherwise,

Take the Complement of the given Arch or Angle to 90, and find the Sine of half thereof, by Prop. 1. Double the Sine thus found, and you have the Co-Chord requir'd.

P R O P. XII.

The Radius and Sine of an Arch or Angle being given, To find the Arch or Angle.

The

The RULE.

First find the Co-S. by Prop. 3. Then consider which is the greatest, the S. or the Co-S. For if the S. be the { Greater, } then the Arch or Angle sought is { Greater } than 45 Degrees. Then if the { S. } be the greater, say, as R. added to half the { S. } is to the { Co-S. }, so is 86, to the { Comp. of the L. sou. Arch or L. sought. }

Example.

If the R. be 1000, and the Sine be 342. What is the Arch or Angle? Answ. 20 Degrees.

P R O P. XIII.

The Radius and Co-Sine of an Arch or Angle being given; to find the Arch or Angle.

The RULE.

First find the Sine by Prop. 2. Then find (by Prop. 12.) the Arch, or Angle answering to that Sine; and so you have what you want. This needs no Example.

P R O P. XIV.

The Radius and Tangent of an Arch, or Angle being given; To find the Arch, or Angle.

The RULE.

First find two Numbers, the Z. of whose Squares is equal to the Square of the R, and whose Proportion to each other, is as the R. to the given T; which two Numbers are found thus: — Say, as the Z. of the squares of the Ratio's, is to the Z. of the squares of the Numbers (equal to the square of the R.); so is the square of the { Greater } Ratio, to the square of the { Lesser }

R

Greater

{ Greater } Number. Which two Numbers are the
 { Lesser } Sine, and Co-Sine.

Or,

(Which is the same thing) say, as the square of the R. added to the square of the given T. is to the square of the R. so is the square of the { Radius }
 { given T. }, to the square of the { S. or Co-S. }
 { Co-S. or S. }. Having thus found the S. and Co-S. Find the Arch or Angle, by Prop. 12.

Or more briefly thus :

First find the Secant, thus ; To the square of the R. add the square of the T. the square Root of the Sum is the Secant. Then (if the given T. be { Less }
 { Greater } than the R.) say, as the Sec. added to $\frac{1}{2}$ the { R. }
 { T. }, is to the { T. }
 { R. }, so is 86, to the { Arch or Angle, sought. }
 { Compl. of the Arch or L. }

P R O P. XV.

The Radius and Co-Tangent of an Arch or Angle, being given ; To find the Arch or Angle.

The R U L E.

First find the T. thus ; say, as the Co-T : R :: R : T ; Then find the Arch or Angle, by Prop. 14.

P R O P. XVI.

The Radius and Secant of an Arch or Angle being given ; To find the Arch, or Angle.

The R U L E.

First find the Co-S. thus ; say, as the Se : R :: R : Co-S. Then find the Arch, or Angle, by Prop. 13.

Other-

Otherwise.

First find the T . thus ; from the Square of the Se . Subtract the Square of the R . the Square Root of the Remainder is the T . Then find the Arch or Angle by Prop. 14.

P R O P. XVII.

The Radius and Co-Secant of an Arch or Angle being given ; To find the Arch or Angle.

The R U L E.

First find the Sine, thus ; say, as the $Co-Se : R :: R : S$. Then find the Arch or Angle, by Prop. 12.

P R O P. XVIII.

The Radius and Versed-Sine of an Arch or Angle being given ; To find the Arch or Angle.

The R U L E.

First find the Co-Sine, thus ; Subtract the $V-S$ from the R . the Remainder is the $Co-S$. Then find the Arch or Angle by Prop. 12.

P R O P. XIX.

The Radius and Co-Versed-Sine of an Arch or Angle being given ; To find the Arch or Angle.

The R U L E.

First find the Sine, thus ; Subtract the $Co-V-S$ from the R . the Remainder is the S . Then find the Arch or Angle, by Prop. 12.

P R O P. XX.

The Radius and Chord of an Arch or Angle being given ; To find the Arch or Angle.

The R U L E.

First find the Sine of half the Arch, or Angle, thus ; Half the Chord is the Sine. Then find the Arch

or Angle by Prop. 12. Double the Arch or Angle thus found, and you have the Arch or Angle sought.

P R O P. XXI.

*The Radius and Co-Chord of an Arch or Angle being given;
To find the Arch or Angle.*

The R U L E,

Find the Arch or Angle by Prop. 20. Then take its Complement to 90 Degree; and you have what you sought.

P R O P. XXII.

*To solve any Question in Trigonometry, Plain, or Spherical,
(or in Astronomy, Geography, Navigation, and Dialing),
without the help of Tables of Sines, Tangents, Secants, &c.*

The R U L E.

Knowing the Analogy by which the Question is to be wrought: First assume the R. to be 1, 10, 100, or 1000 &c. Then find the Sines, or Tangents, &c. (of the Degrees that come in Questions) by some of the first precedent Propositions, for that Purpose: Then work the Analogy by the Line; which being done, find (by some of the last 10 precedent Propositions) the Arch or Angle answering to the S. or T. &c. found; and the Question is Solved.

Example.

I would find the Altitude of the Sun at the Hour of 6. (in the Lat. of 51 Deg.) when the Sun's Declination is 10 Deg. The Analogy to do it, is, as R: S, Lat.: S. Sun's Declination: S. Altitude. I assume the R to be 100. Then (by Prop. 1.) I find the S. of 51 Deg. the Lat. to be 76, and the S. of 10 Deg. the Declination to be 17. Then I say, as 100: 76:: 17: 13, which S, 13. I turn into Deg. by Prop. 12. and I find it to be

$7\frac{1}{2}$ Deg. *Fere*, which is the Altitude sought. Do the like by any other Question.

C H A P. II.

Of Right-Angl'd Plain Trigonometry.

IN Right-Angl'd Plain Trigonometry, there is commonly reckon'd but 7 different Cases; I shall make 15, (distinguish'd by their different *Data* and *Quæsitæ*, and shall shew several Rules for the Solution of most of them;) all which (for Brevety Sake) I shall lay down in a brief Synopsis, as is commonly done in Mathematical Books; but my Synopsis will be somewhat different from theirs, because I am willing (in this Book) to avoid the Charge of Cuts. But before I proceed, the Reader may

Note: 1. That in Right-Angl'd plain Triangles, if one of the Oblique Angles be given, the other also is given, as being its Complement to 90 Degrees.

2. That in a Right-Angl'd plain Triangle, any of its sides containing the Right-Angle, may be the Base; though commonly the longest of those sides is call'd the Base, and the other Perpendicular.

3. That in the following Synopsis, I shall (for Brevity-sake) make use of the following Symbols, viz.

R. Radius.	B. Base.
S. Sine.	P. Perpendicular.
T. Tangent.	H. Hypotenuse.
Co-S. Co-Sine.	A. Angle.
Co-T. Co-Tangent.	L B. Angle at the Base.
Sec. Secant.	L P. Angle at the Perpendicular.

This being premis'd, I here Insert

The Cases of Right Angl'd Plain Triangles.

Case	Given	Requir'd	The Rule.
I	Ls $\angle$ B	P	1. $As S. LP : B :: S. LB : P.$ or 2. $As R : B :: T. LB : P.$ or 3. $As T. LP : B :: R : P.$
II	L & m	H	1. $As S. LP : B :: R : H.$ or 2. $As R : B :: Sec. LB : H.$ or 3. $As T. LP : B :: Sec. LP : H.$
III	Ls $\angle$ P	B	1. $As S. LB : P :: S. LP : B.$ or 2. $As R : P :: T. LP : B.$ or 4. $As T. LB : P :: R : B.$
IV	$Idem$	H	1. $As S. LB : P :: S. LP : B.$ or 2. $As R : P :: Sec. LP : B.$ or 3. $As T. LB : P :: R : B.$
V	Ls $\angle$ H	B	1. $As R : H :: S. LP : B.$ or 2. $As Sec. LB : H :: R : B.$ or 3. $As Sec. LP : H :: T. LP : B.$
VI	$Idem$	P	1. $As R : H :: S. LB : P.$ or 2. $As Sec. LP : H :: R : P.$ or 3. $As Sec. LB : H :: T. LB : P.$
VII	B & P	LB	$As B : R :: P : T. LB.$
VIII	$Idem$	LP	$As P : R :: B : T. LP.$
IX	$Idem$	H	Find $LP.$ by the 8th Case, then say, $As S. LP : B :: R : H.$
X	B & H	LB	$As B : R :: H : Sec. LB.$
XI	$Idem$	LP	$As H : R :: B : Sec. LP.$
XII	$Idem$	P	Find $LB.$ by the 10th Case, then say, $As R : B :: T. LB : P.$
XIII	P & H	LB	$As H : R :: P : Sec. LP.$
XIV	$Idem$	LP	$As P : R :: H : Sec. LP.$
XV	$Idem$	B	Find $LP.$ by the 4th Case, then say, $As R : P :: T. LP : B.$

The Explanation of the Synopsis, by

Example.

Suppose in a Right-Angl'd Triangle, there were given both the Oblique Angles, and the Base; to find the Hypotenuse. This is the second Case; and the fourth Column of the Synopsis shews three Rules to solve it: the first of which is to be read thus; As the Sine of the Angle at the Perpendicular, is to the Base; so is the Radius to the Hypotenuse. The rest are to be read in the same manner. The thing is too plain to need any farther Explanation. I come therefore to

C H A P. III.

Of Oblique Angl'd Plain Trigonometry.

THis I shall lay down in a brief Synopsis. But first you may

Note: 1. That in all Oblique Angl'd Triangle, if two Angles are given, the third Angle is also given, as being the Complement of their Sum to 180 Degrees.

2. That here I shall make use of some Symbols not explain'd in the 2d Chapter, and they are these, *viz.*

L. (<i>Latus</i>) a Side.	X Difference.
l. Sides.	— Greater.
L. Angle.	— Lesser.
l. Angles.	= Equal.
Δ. Triangle.	+ Added.
Σ. Sum.	— Subtracted.

This being Pre-nis'd, I shall now lay down

The

The Case of Oblique Angled Plane Triangles.

Case	Given	Requir'd	The Rule.
I	2Ls & L. includ.	the other Ls.	As S. 3d L : given L :: S. L opposite to L. sought : L sought.
II	2Ls & L. opposite to one of 'em.	the L. opposite to the other.	As S. L opposite to the given L : the given L :: S. L opposite to L. sought : the L. sought.
III	Idem.	L. includ.	As S. L opposite to the given L : given L :: S. 3d L : L. sought.
IV	2Ls & L includ.	the other Ls.	As Z. given Ls : their X :: T. $\frac{1}{2}$ Z. of the other 2Ls : T. $\frac{1}{2}$ their X. which $\frac{1}{2}$ Differ. { Add. to } the $\frac{1}{2}$ Z. is = { Sub. from } $\left\{ \begin{array}{l} \square \\ \square \end{array} \right\}$ L sought.
V	Idem.	the 3d L.	Find the other Ls. by the 4th Case. Then find the 3d L. by the 2d Case.
VI	2Ls & L. opposite to one of 'em.	L opposite to the other.	As one L : S. its opposite L :: the other L : S. its opposite L.
VII	Idem.	L includ.	Find the other L by the 6th Case. Then find L sought by Note 1. of this Chapter.
VIII	Idem.	the 3d L.	Find L includ., by the 7th Case. Then find L. sought by the 2d Case.

Cafe	Given	Requir'd	The Rule.
IX	3 L s.	an L .	Divide the Oblique $Ld \Delta$ into two right Ld ones, by letting fall a P . from the Obtuse L . to the $\perp L$. thus: say, as the $\perp L$: Z . other 2 L s:: X . of them 2 L s: a 4th. Which 4th Sub. from $\perp L$. the P . will fall in the middle of the Remainder. Then in each of the two right $Ld \Delta$ s is given the B . and H . Whence the L sought may be found by the 6th Cafe of right Ld Plain Trigonometry.

Thus I have exhibited eight Cafes in Right-Angl'd, and 4 in Oblique-Angl'd Trigonometry, more than any other Author.

This is so plain that it needs no farther Explanation: I shall therefore proceed to

CHAP. IV.

Of Right-Angl'd Spherical Trigonometry.

I shall here also lay down the Rules for the Solution of

The

The Cases of Right Angled Spheric Triangles.

Case	Given	Requ.	The Rule.
I	Ls at $P \& B$	H	$As\ t.LP:Co-t.LB::R.Co-S.H.$ or $As\ R:Co-t.LB::Co-t.LP:Co-S.H.$
II	Idem	B	$As\ S.LB:R::Co-S.LP:Co-S.P.$
III	Idem	P	$As\ S.LP:R::Co-S.LB:Co-S.P.$
IV	$H \& LB$	LP	$As\ R:Co-S.H::t.LB:Co-t.LP.$
V	Idem	P	$As\ R:S.H::S.LB:S.P.$
VI	Idem	B	$As\ R:Co-S.LB::t.H:t.B.$
VII	$H \& LP$	LB	$As\ R:Co-S.H::t.LP:Co-t.LB.$
VIII	Idem	B	$As\ R:S.H::S.LP:S.B.$
IX	Idem	P	$As\ R:Co-S.LP::t.H:t.P.$
X	$B \& LB$	LP	$As\ R:Co-S.B::S.LB:Co-S.LP.$
XI	Idem	H	$As\ R:Co-S.LB::Co-t.B:Co-t.H.$
XII	Idem	P	$As\ R:S.B::t.LB:t.P.$
XIII	$B \& LP$	LB	$As\ R:Co-S.B::S.LP:Co-S.LB.$
XIV	Idem	H	$As\ R:Co-S.LP::Co-t.B:Co-t.H.$
XV	Idem	P	$As\ R:Co-t.LP::t.B:S.P.$
XVI	$P \& LB$	LP	$As\ Co-S.P:R::Co-S.LB:S.LP.$
XVII	Idem	H	$As\ S.LB:R::S.P:S.H.$
XVIII	Idem	B	$As\ t.LB:t.P::R:S.B.$ or $As\ R:Co-t.LB::t.P:S.B.$
XIX	$B \& LP$	LB	$As\ R:Co-S.P::S.LP:Co-S.LB.$
XX	Idem	H	$As\ R:Co-S.LP::Co-t.P:Co-t.H.$
XXI	Idem	B	$As\ R:S.P::t.LP:t.B.$
XXII	$H \& B$	LP	$As\ S.H:R::S.B:S.LP.$
XXIII	Idem	LB	$As\ R:t.B::Co-t.H:Co-S.LB.$
XXIV	Idem	P	$As\ Co-S.B:R::Co-S.H:Co-S.P.$
XXV	$H \& P$	LB	$As\ S.H:R::S.P:S.LB.$

Cafe	Given	Requ.	The Rules.
XXVI	Idem	LP	As $R : t.P :: Co-s. H : Co-S. LP.$
XXVII	Idem	B	As $Co-S. P : R :: Co-S. H : Co-S. B.$
XXVIII	B & P	LB	As $R : S. B :: Co-t. P : Co-t. LB$
XXIX	Idem	LP	As $R : S. P :: Co-t. B : Co-t. LP.$
XXX	Idem	H	As $R : Co-S. P :: Co-S. B : Co-S. H.$

These are all the possible Cafes of Right-Angl'd Spherical Trigonometry; and here are 14 Cafes more than in any other Author.

CHAP. V.

Of Oblique-Angl'd Spherical Trigonometry.

Here also, I shall lay down the Rules for the Solution of

The Cafes of Oblique Angl'd Sph. Triangles.

Cafe	Given	Requir'd.	The Rule.
I	2 L s. and Z . op. to one of them.	L . oppo- site to the other.	As $S. L$. opposite to the given $L : S$. given $L :: S. L$. opposite to L . sought : $S. L$. sought.
II	Idem.	L . be- tween them.	Find L . opposite to the o- ther given L . by 1st Case. Then say, as $S. \frac{1}{2} X$ given $LS :$ $S. \frac{1}{2}$ their $Z :: T. \frac{1}{2} X$ given $LS : T. \frac{1}{2} L$ sought.

Cafe

Cafe	Given	Requir'd	The Rule.
III	Idem.	the 3d. L .	Find L . included by 2d Cafe. Th. say, as S . giv. L : S . L . op. to it : LS . L . last found : S . L . fou.
IV	2 LS . is L . included	the 3d. L .	As Co . 1. of 1 of the giv. LS : R : Co . 3. giv. L : Co . 1. a 4th. Then if 2d giv. L . be { Acute, 2 } { Obtuse X } of this 4th & 2d giv. L . is a 5th Then, as S . 4th : S . 5th : Co . 3. first giv. L : Co . 3. L . sought.
V	Idem.	one of the other LS .	Find 3d L . by 4th Cafe. Then, say, as S . 3d L : S . giv. L : S . L . op. to L . sought : S . L . fou.
VI	2 LS & L . op. to one of them.	L . opposite to the other.	As S . L . opposite to the giv. L : S . given L : S . L . opposite to L . sought : S . L . sought.
VII	Idem.	the L . included.	As R : Co . 3. L . adj. to the giv. L : T . giv. L : Co . 1. a 4th. Again, as Co . 1. of the same L : Co . 1. of the oth. giv. L : Co . 3. the 4th : Co . 3. a 5th. If the giv. L . be { Ac. 2 } { Ob. x } of the 4th & 5th is L . sought.
VIII	Idem.	the 3d. L .	As R : Co . 3. giv. L : T . giv. L adj. to the giv. L : a 4th. Again, as Co . 3. of the same L : Co . 3. the oth. giv. L : Co . 3. the 4th : Co . 3. a 5th. Then if the given L . be { Ac. 2 } { Ob. x } of the 4th & 5th is L . sought.

Case	Given	Requir'd	The Rule.
IX	2 <i>Ls.</i> & <i>L</i> included.	any of the other <i>Ls.</i>	As $R : Co-S.$ given $\angle :: \angle$ given L opposite to $\angle$ sought : $t.$ a 4th. Then, if the given $\angle$ be { Obtuse, z } of Acute, x } of this fourth and the other given L is a fifth. Then, as $S. 4th : S. 4th :: t.$ given $\angle$: $\angle$ sought.
X	<i>Idem.</i>	3d <i>L.</i>	Find the other <i>Ls.</i> by 9th Case. Then find the third <i>L.</i> by 1st Case.
XI	3 <i>Ls.</i>	any one of the <i>Ls.</i>	As the Rectangle of the $Ss.$ <i>Ls.</i> containing $\angle$ req; Square of $R ::$ Rectangle of the $Ss.$ $Ns.$ of the same <i>Ls.</i> from $\frac{1}{2} z$ of the 3 <i>Ls.</i> : Square $S.$ of $\frac{1}{2} \angle$ sought.
XII	3 <i>Ls.</i>	any one of the <i>Ls.</i>	Turn <i>Ls.</i> into $Ls.$ And then Work as in the 11th Case.

Note : There are 6 Cases in Oblique-Angl'd Spherical Trigonometry that are call'd doubtful; because, unless the Affection of the side, or Angle sought, (*viz.* whether it be greater or lesser than 90 Deg.) be also given, it is uncertain whether you have found the Side or Angle sought, or its Complement to 180 Degrees. These doubtful Cases are here the 1, 2, 3, 6, 7. and 8. The Reader may see more of this Matter, (with a full Demonstration of it,) in *Clavius Theodosium*. Or if the Reader be only an English Scholar, he may see the same Translated and Inserted by Mr. Collins, in his *Sector* on the *Quadrant*, or Transcribed from Mr. Collins (*Verbatim*) by Mr. Holwel, in his *Trigonometry made easy*. To some of these Authors I refer the Reader; it being not my

S

Busi-

Business here to teach whole Sciences; but only to shew the Excellency and Usefulness of the *Line of Numbers* in each.

CHAP. VI.

Of Astronomy.

FOR Brevity Sake, I shall dispose (according to my usual Method) the several Problems of this Chapter, in a brief Compendium; which yet I doubt not, but to render as plain and intelligible, as those that have said ten times as much. But before I proceed further, it will be convenient to explain the Symbols there made use of; and they are these,

R. Radius.	<i>Æq. P.</i> Æquinoctial-Point.
S. Sine.	<i>L-⊙-Pos.</i> Ang. of $\odot$ Posit.
Co-S. Co-Sine.	H. Hour.
T. Tangent.	D. Day.
Co-t. Co-Tangent.	Ni. Night.
V-S. Versed-Sine.	M-D. Mid-Day.
Sec. Secant.	M-N. Mid-Night.
$\odot$ Sun.	T. Time.
$\lrcorner$ Moon.	T-D. Time of Day.
* Star.	T-N. Time of Night.
Ln. Longitude.	E. East.
Lt. Latitude.	W. West.
Lt-Pl. Latitude of a Place.	N. North.
Dcl. Declination.	S. South.
R-A. Right Ascension.	$\sqsupset$ Greater, or Greatest.
O-A. Oblique Ascension.	$\sqsubset$ Lesser or Least.
O-D. Oblique Descension.	Z. Sum.
A-X. Ascensional Differ.	X. Difference.
Az. Azimuth.	+ Added.
Am. Amplitude	- Subtracted.
Al. Altitude.	

This

This being Premised, I here exhibit the
Problems in Astronomy

Prob.	to find.	knowing.	The Rule.
1	☉ Dcl.	☉. <i>Ln.</i> from γ or $\triangle$.	As $R : S$. ☉. $\square$ Dcl : : S . ☉. <i>Ln</i> from the next <i>Æq.p.</i> (either γ or $\triangle$) : S . ☉ Dcl.
2	☉. <i>Ln.</i> from γ or $\triangle$	its present Dcl.	As S . ☉ $\square$ Dcl : S . its present Dcl. : : $R : S$. ☉. <i>Ln</i> from the next <i>Æq.p.</i> either γ or $\triangle$.
3	Idem.	its $R-A$.	As $R : Co-t$. ☉. $R-A$: : $Co-S$. its $\square$ Dcl : $Co-t$. its <i>Ln.</i> from the next <i>Æq.p.</i>
4	$A-X$. ☉ or *	<i>Lt-Pl.</i> and ☉ or * Dcl.	As $Co-t$. <i>Lt-Pl</i> : t . ☉ or * Dcl : : $R : S$. his $A-X$.
5	☉ <i>Am.</i>	<i>Lt-Pl.</i> and ☉ <i>Ln.</i> from γ or $\triangle$	As $Co-S$. <i>Lt-Pl</i> : S . ☉. $\square$ Dcl. : : S . its <i>Ln.</i> from γ or $\triangle$: : S . its <i>Am.</i>
6	<i>Am.</i> ☉ or *	<i>Lt-Pl.</i> and ☉ or * Dcl.	As $Co-S$. <i>Lt-Pl</i> : R : : S . ☉ or * Dcl : S . its <i>Am.</i> from E . or W .
7	☉ $R-A$	its <i>Ln.</i> from γ or $\triangle$	As $R : Co-S$. ☉. $\square$ Dcl : : t . its <i>Ln.</i> from γ . or $\triangle$: : t . its $R-A$.
8	Idem.	its Dcl.	As t . ☉. $\square$ Dcl : t . its present Dcl : : $R : S$. its $R-A$.

Prob.	to find	knowing.	The Rule.
9	R-A. of a Planet or fix- ed *.	its Ln. and Lt.	As $R : S^* . Ln.$ from V or ∞ :: $Co-S.$ its $Lt : t.$ a 4th Arch. Then, if the *. $Ln.$ and $Lt.$ be { alike, (viz. b. $N.$ or b. $S.$) x. } { unlike, (viz. i. $N.$ & i. $S.$) z. } of this 4th Arch, and $\odot$ greater Dcl. is a 5th Arch. Then, as $S.$ 4th Arch : $S.$ 5th : $t.$ *. $Ln : t.$ its R-A.
10	O-A of $\odot$ or *.	its R-A and A-X.	Find A-X. by Prob. 4. and R-A. by Prob. 7, 8, or 9. Then if Dcl. be { $N. x$ } R-A and { $S. z$ } A-X is O-A.
11	O-D. of $\odot$ or *.	its R-A and A-X.	Find A-X. by Prob. 4. and R-A. by Prob. 7, 8, or 9. Then if Dcl. be { $N. z$ } of R-A { $S. x$ } and A-X is O-D.
12	Dcl. of a Pla- net or Fixed *.	its Ln. and Lt.	Find a 4th and 5th Arch, by Prob. 9. Then say, as $Co-S.$ 4th Arch : $Co-S.$ 5th : $S.$ *. $Lt : S.$ its Dcl.
13	L- $\odot$: Pos. at its Rif.	Lt-Pl. and $\odot$. Dcl.	As $Co-S.$ $\odot$. Dcl : R :: $S.$ Lt- Pl : $S.$ L- $\odot$ -Pos. at T. of its Rising.
14	L- $\odot$ P. at any T-D.	Lt-Pl. and T-D.	As $R : Co-S.$ Lt-Pl : $S.$ T. from M-N : $S.$ L- $\odot$ -Pos. at that T.
15	Eclip. and Merid.	$\odot$. Great Dcl. and R-A.	As $R : S.$ $\odot$ Greater Dcl : $Co-$ $S.$ $\odot$. R-A : $Co-S.$ L. of the Ecliptic and Meridian.

Prob.	to find	knowing.	The Rule.
16	☉ <i>Al.</i> at <i>E.</i> or <i>W.</i>	<i>Lt-Pl.</i> and ☉. <i>Dcl.</i>	<i>As S. Lt-Pl : S. Sun's Dcl :</i> <i>R : S. its Al. when it is due</i> <i>E. or W.</i>
17	☉ <i>Al.</i> at fix a Clock.	<i>Lt Pl.</i> and <i>Sun's Dcl.</i>	<i>As R : S. Sun's Dcl :: S. Lt. Pl :</i> <i>S. Sun's Al. at fix a Clock.</i>
18	☉ <i>Az.</i> at fix a Clock.	<i>Lt-Pl.</i> and <i>Sun's Dcl.</i>	<i>As Co-S. Lt-Pl : R :: Co-t.</i> <i>Sun's Dcl : t. its Az. (from the</i> <i>N.) at fix a Clock.</i>
19	Meri- dian <i>Al.</i> ☉ or *	<i>Lt-Pl.</i> and <i>Sun</i> or * <i>Dcl.</i>	If the <i>Lt-Pl.</i> & <i>Sun's</i> or * <i>Dcl.</i> be { alike, (viz. b. <i>N.</i> or b. <i>S.</i>) } <i>z</i> { unlike, (viz. i. <i>N.</i> & i. <i>S.</i>) } <i>x</i> of <i>Sun</i> or * <i>Dcl.</i> & <i>Com. Lt Pl.</i> is the Meridian <i>Al.</i> of <i>Sun</i> or *.
20	☉ <i>Al.</i> at any <i>I-D.</i>	Length of an Object, & of its Shadow.	As the length of the Sha- dow (of any Object set Per- pendicular to the Horizon) to the length of the Object ; so is <i>R.</i> to the <i>t. Sun's Al.</i>
21	Idem.	<i>Lt-Pl.</i> and <i>Sun's Dcl.</i> and <i>I-D.</i>	<i>As R : Co-S. T. from M-D : :</i> <i>Co-t. Lt-Pl : t. a 4th Arch.</i> Which 4th Arch Sub. from the <i>Sun's Dist.</i> from the Pole leaves a 5th Arch. Then, as <i>Co-S. 4th</i> <i>Arch : Co-S. 5th :: S. Lt-Pl :</i> <i>S. Sun's Al. at that T.</i>
22	the same another way.	the same.	<i>As R : Co-S. Lt-Pl :: Co-S. the</i> <i>Suns Dcl : a 4th S.</i> Again, as <i>R :</i> that 4th <i>S : : S. T. from S a Cl.</i> to (in Summer) <i>X. Ss. Suns Al.</i> at <i>S.</i> and of its <i>Al. sou.</i> and (in Winter) to <i>Z. Ss. its depression</i> at <i>S.</i> and of its <i>Al. sought.</i>

Prob. to find.	knowing.	The Rule.
23	the <i>Sun's</i> <i>Alt.</i> on any <i>Az.</i> $\odot$ be. in the Equin.	As the Cube of <i>R</i> : double Rect-angle of the <i>Co-Ss.</i> of <i>Lt.Pl.</i> & $\odot$ <i>Del</i> : : square <i>S.</i> $\frac{1}{4}$ <i>T.</i> from <i>M-D</i> : <i>X.</i> <i>Ss.</i> $\odot$ Meridian <i>Al.</i> & <i>Al.</i> sought. (The Merid. <i>Al.</i> is found by Prob. 19.)
24	<i>Sun's</i> <i>Alt.</i> on any <i>Az.</i> $\odot$ be. in the Equin.	As <i>R</i> : <i>Co-S.</i> the <i>Sun's</i> <i>Az.</i> fr. South : : <i>Co-t.</i> <i>Er-Pl</i> : <i>t.</i> <i>Sun's</i> <i>Al.</i> upon that <i>Az.</i> (the <i>Sun</i> being in the Equinox.
25	$\odot$ <i>Al.</i> upon any <i>Az.</i> at any <i>T.</i>	Find the <i>Sun's</i> <i>Al.</i> up. that <i>Az.</i> in the Equin. by Prob. 24. Then, say, as <i>S.</i> <i>Lt-Pl</i> : <i>S.</i> <i>Sun's</i> <i>Del</i> : : <i>Co-S.</i> <i>Sun's</i> <i>Al.</i> on th. <i>Az.</i> in the Equi. <i>S.</i> a 4th Arch. Then, if the <i>Lt-Pl.</i> & the <i>Sun's</i> <i>Del.</i> be alike, (viz. b. N. or b. S.) $\times$ } } unlike, (viz. 1 N. & 1 S.) $\times$ } of 4th Arch, and the <i>Sun's</i> <i>Al.</i> in the Equinox is <i>Al.</i> sought. But in all <i>Az's.</i> betwixt the N. and E. or N. and W. use <i>Z.</i> for <i>X.</i> and ϵ <i>Contra.</i>
26	$\odot$ <i>Az.</i> at any <i>T.D.</i>	As <i>Co-S.</i> the <i>Sun's</i> <i>Al</i> : <i>S.</i> <i>T.</i> from <i>M-D</i> : : <i>Co-S.</i> the <i>Sun's</i> <i>Del</i> : <i>S.</i> its <i>Az.</i> from the N.
27	<i>Az.</i> $\odot$ cr * at any <i>T.</i>	As <i>R</i> : <i>Co-S.</i> <i>Lt-Pl</i> : : <i>Co-S.</i> <i>Al</i> : a 4th <i>S.</i> Again, as the 4th <i>S.</i> : <i>S.</i> $\frac{1}{2}$ <i>Z.</i> of the Complem. of the 3 <i>Data</i> : : <i>S.</i> <i>X.</i> that $\frac{1}{2}$ <i>Z.</i> & <i>Com.</i> <i>Del</i> : a 7th <i>S.</i> Which multipl. by <i>R.</i> the square Root of the Product is <i>S.</i> $\frac{1}{2}$ <i>Az.</i> fr. South.

Prob.

Prob.	to find	knowing	The Rule.
28	the same another way.	the same.	As $R : S, Lt. Pl. :: S. Al : a 4th S.$ Again, as $Co-S. Lt-Pl. : Sec. Al ::$ so (if Dcl. be towards the {elevated } {depressed } Pole) is { X } { Z } of the 4th S. and S. Dcl : S. Az. from E. or W.
29	the same another way.	the same.	As the Rectangle of the Ss. $Co-Lt-Pl.$ and $Co-Al$: square $R ::$ Rectangle Ss. $Xs.$ of $Co-Lt-Pl.$ & $Co-Al.$ from $\frac{1}{2} Z.$ $Co-Lt-Pl.$ $Co-Al.$ and $Co-Dcl$: square $S.$ $\frac{1}{2} Az.$ from the N.
30	T. ☉ com.to E. or W	Lt-Pl. & the Sun's Dcl.	As 1. $Lt-Pl : t.$ the Sun's Dcl :: $R : Co-S. T.$ from M-D.
31	the Sun's Rising and Set.	Lt-Pl. & the Sun's Dcl.	Find the Sun's $A-X.$ by Prob. 4. Then, $A-X$ { Sub. fr. } { Add. to } 6 Ho- when ☉. is in $V \delta II \delta \Omega$ or γ gives ☉ { Rising. } { Setting. } And the same { Add. to } { Sub. fr. } 6 Hours, when ☉ is in $\delta m T \gamma$ or $X.$ gives ☉ { Rising. } { Setting.
32	☉ Semi-Di. Ar. & length of D.	Lt-Pl. & the Sun's Dcl.	Find the $T.$ of the Sun's setting by Prob. 31. for that is the Sun's Semi-Diurnal-Arch; which doubled gives the Length of the Day.

Prob.

Prob.	to find	knowing	The Rule
33	Time of the Day.	<i>Lt-Pl.</i> ☉ Dcl. and Meridian and present <i>Al.</i>	As $R : Co-S.$ the <i>Suns</i> Dcl :: $Co-S. Lt-Pl :$ a 4th <i>S.</i> Again, as that 4th $S : R :: X. Ss$ of the Meridian and present $Al : V-s. T.$ from <i>M-D.</i> The Meridian <i>Al.</i> is found by Prob. 19.
34	the sa.	<i>Lt-Pl.</i> & the <i>Sun's</i> Dcl. & <i>Al.</i>	Find the <i>Sun</i> Meridian <i>Al.</i> by Prob. 19. and its Semi-Diurnal Arch, by Prob. 32. Then say, as <i>S.</i> Meridian $Al : S.$ present $Al :: S.$ Semi-Diurnal-Arch : $S. T.$ from the <i>Sun</i> Rising, or setting.
35	the same another way	the same.	As $R : Co-S. Lt-Pl :: Co-S. Al :$ a 4th <i>S.</i> Again, as that 4th $S : S \frac{1}{2} z.$ Complements 3 <i>Data :: S. X.</i> that $\frac{1}{2} z.$ from the Compl. $Al : a$ 7th <i>S.</i> which multiplied by <i>R.</i> the square Root of the Product is $S. \frac{1}{2} T.$ fr. Noon.
36	the same another way.	the same.	As $R : S, Lt-Pl :: S. \odot$ Dcl $S. a$ 4th, wch. in { Sumr. ☉. <i>Alt.</i> } { Wint. ☉. <i>Depr.</i> } (at 6 a Clock.) Th. as $Co-S. Dcl :$ $Sec. Lt-Pl ::$ (in { Sum. x } { Win. z } $S's.$ of <i>Al.</i> given & the { <i>Al.</i> } { <i>De.</i> } at 6 a Clock : $S. T.$ from 6 towards <i>M-D.</i> in Winter ; & in Sum. also, when the given <i>Al.</i> is greater than <i>Al.</i> at 6, but when 'tis lesser, towards <i>M-N.</i>

Prob.

Prob.	to find	knowing.	The Rule.
37	Idem	<i>Lt-Pl.</i> the <i>Suns</i> Dcl. & <i>Al.</i> and Semidiur. Arch.	As $R : t. Lt-Pl :: t. \odot$ Dcl : $S.$ 4th. (which 4th is $T. \odot$ Rising from 6 a Clock.) Again, as Rectangle $Co-S. Lt-Pl. \& Co-S.$ the <i>Sun's</i> Dcl : Square $R :: S. Al : X. V-S's.$ of the Semidiurnal Arch and of $T.$ sought.
38	Idem.	the <i>Suns</i> Dcl. <i>Al.</i> and <i>Az.</i>	As $Co-S.$ the <i>Suns</i> Dcl : $S.$ its $Az.$ from $N :: Co-S.$ its $Al : S.$ $T.$ from Noon.
39	<i>Lt-Pl.</i>	the <i>Suns</i> Dcl. and <i>A-X.</i>	As $S.$ the <i>Sun's</i> $A-X : R :: t.$ the <i>Sun's</i> Dcl : $Co-t. Lt-Pl.$
40	Idem.	the <i>Sun</i> or * <i>Am.</i> and Dcl.	As the <i>Suns</i> or * $Am : S.$ its Dcl : $R : Co-S. Lt-Pl.$
41	Idem.	$\odot$. Dcl. & $T.$ at $E.$ or $W.$	As $Co-S. T.$ from Noon : $R :: t.$ the <i>Sun's</i> Dcl : $t. Lt-Pl.$
42	Idem.	$\odot$. Dcl. & <i>Al.</i> when $E.$ or $W.$	As $S.$ the <i>Sun's</i> $Al.$ when $E.$ or $W. : R :: S.$ the <i>Sun's</i> Dcl : $S. Lt-Pl.$
43	Idem.	the <i>Suns</i> Dcl. and <i>Az.</i> at 6.	As $t.$ the <i>Suns</i> $Az.$ at six a Clock : $Co-t.$ its Dcl : $R : Co-S. Lt-Pl.$
44	Idem.	$\odot$. <i>Al.</i> & <i>Az.</i> at any $T-D.$ $\odot$ being in the Equi.	As $Co-S.$ the <i>Sun's</i> $Az.$ from the South (the <i>Sun's</i> being in the Equinox) : $R :: t.$ the <i>Sun's</i> $Al.$ on that $Az : Co-t. Lt-Pl.$
45	$T. \odot$ coming to $S.$	$\odot$ Age.	As $5 : 4 :: \odot$. Age (in Days) : $T. \odot$ coming to $S.$ (in Hours Afternoon.)

Prob.	to find	knowing.	The Rule.
46	(Set. in the Increa. & Rif. in the Decrease.	(Age.	As 5:4:: so in the {Incr. } {Decr. } is the { (Age (in Days) } 30 less (Age. } to the T. (in Hours) that the { Sets after } the Sun. { Rises before }
47	T-N. by the (.	(Age.	Find the (Southing by Prob. 45. Then (by the shadow cast on a Sun-Dial) see how much it wants of, or is past the S. which T. subtr. from, or added to the Southing gives T-N:
48	T. of any Pl. or fixt * coming to the S.	its In. and Lt.	Find the Sun's R-A by Prob. 7. or 8 and * R-A. by Prob. 9. Then, if the Sun's R-A Subtr. from * R-A. (adding 360 Deg. or 24 Ho. to * R-A. if it be the lesser than the Sun R-A) leaves T. *'s coming to °.
49	Rising or Set. of any Planet fix'd *.	It-Pl. and * Decl. and T. of its Southng.	Find * A-X. by Prob. 4. Then, A-X. { Add. to } { Sub. fr. } 6 Ho. when Decl. is { N. } { S. } gives an Arch, which { Add. to } { Subt. fr. } T. *. Sout. gives T. its { Setting. } { Rising. }

Prob.	to find	knowing	The Rule.
50	T.N. by the Plan. * or fixt Stars.	Lt-Pl. & * Del. & Al.	Find * Meridian <i>Al.</i> by Prob. 19. and <i>T.</i> its South- ing by Prob. 48. Then say, As $R : Co-S. * . Del :: Co-S.$ $Lt-Pl : a 4th S. Again, as$ that $4th S : R :: X. S's. *$ Meridian and Present <i>Al.</i> $V-S. *$ Distance (in <i>T.</i>) from the Meridian. There- fore, if <i>T.</i> thus found be { Added to } <i>T. * South-</i> { Subtr. from } ing, when * is in { <i>W. Sum.</i> } is <i>T-Night.</i> { <i>E. Rem.</i> }
51	the same ano- ther way.	the same.	Find * Southing, by Prob. 48. Then say, as $R : Co-S.$ $Lt-Pl :: Co-S. * . Al : a 4th$ $S. Again, as this 4th S : S.$ $\frac{1}{2} Z$ Complements of the 3 <i>Data :: S. X. of that $\frac{1}{2} Z$</i> from the Compl. <i>Al : a 7th S.</i> which multiplied by <i>R.</i> the Square Root of the Product is $S. \frac{1}{2} T. *$ wants of, or is past the Meridian. And this <i>T.</i> { Added to } { Subtracted from } <i>T. * Southing,</i> gives the Time of the Night.

The Use of the foregoing Compendium,

Is very easy. For in the first Column, the Problems
are Numbered ; the second, exhibits what is requir'd ;
the third, what is given ; and the fourth, shews the
Rule by which each Problem is solved. So that, the first
Column

Column shews the Number of each Problem; the second and third, its Title; and the 4th, the Rule for its Solution.

Example.

The first Problem is to be read thus, *Problem the first. To find the Sun's Declination, by knowing its Longitude from Aries or Libra. The Rule. As Radius, to the Sine of the Sun's greatest Declination; so is the Sine of the Sun's Longitude from the next Equinoctial Point (either γ or ϖ), to the Sine of the Sun's Declination.*

In the same manner are all the rest of the Problems to be read. I presume the thing is too plain to need another Example: If any thing seem difficult, it is by reason of the Symbols; which yet are so naturally adapted to the things they are to signify, that any one (tho' but of an ordinary Capacity) may easily learn them all in half an Hour.

I shall therefore only add a few Notes, upon some of the Problems, and so conclude this Chapter.

Notes upon

Prob. 1. The $\odot$. greatest Decl. is always 23 Degrees and 29 Minutes.

Prob. 10. This Rule is made for N. Latitudes. In S. Latitudes, if Decl. be $\left\{ \begin{smallmatrix} N. z \\ S. x \end{smallmatrix} \right\}$ of R-A and A-X is 0-4.

Prob. 11. In S. Latitudes work as is directed in the Note on Prob. 10.

Prob. 31. In S. Latitudes use Addition for Subtraction, and the contrary.

Prob. 49. In S. Latitudes, the A-X must be $\left\{ \begin{smallmatrix} + \\ - \end{smallmatrix} \right\}$

when the Decl. is $\left\{ \begin{smallmatrix} S. \\ N. \end{smallmatrix} \right\}$.

CHAP. VII.

Of Geography.

THe Principal and most usual Problems in Geography, are those that follow.

P R O B. 1.

To reduce Great Circle Degrees (as of the Equinoct, or Meridian, &c.) into English Miles.

The RULE.

Say, as 1 : 70 :: the given Number of Degrees : the Miles answering thereto.

P R O B. 2.

To reduce the Degrees of any Parallel of Latitude, into English Miles.

The RULE.

As R to Co. S. Lt. of the Parallel : so is any Number of Degrees in that Parallel, to the Great Circle Degrees answering thereto. Which turn'd into Miles (by Prob. 1.) gives the Answer.

P R O B. 3.

Two Places which differ only in Lt. To find their Distance.

The RULE.

If the two Places lye { both N. or both S. } of the Equinox, { x. } of their Latitudes turn'd into Miles, (by Prob. 1.) gives their Distance.

P R O B. 4.

Two Places which differ only in Lng. To find their Distance.

T

The

The RULE.

1. If the two Places lye both under the Equinox, their $X\text{ }Lng$ (if it be less than 180 Deg.) turn'd into Miles, (by Prob. 1.) gives their Distance. But if their $X\text{ }Lng$ exceed 180 Degrees, take its Complement to 360 Deg. and work as before.

2. If the two places lye (not under the Equinox, but) under some Parallel of Lt . Say, as $R : Co-S$ common $lt : \frac{1}{2} X\text{ }Lng : \frac{1}{2}$ Distance in Degrees, which turn'd into Miles, (by Prob. 1.) gives the Distance sought.

PROB. 5.

Two Places differing both in $Long$. and Lat . To find their Distance.

The RULE.

1. If one of the Places lye under the Equinox, and the other under some Parallel of Lt . Say, as $R : Co-S$. of their $X\text{ }Ing : Co-S. Lt. Co-S.$ their Distance in Deg. which turn'd into Miles (by Prob. 1.) gives the Answer.

2. If both Places lye without the Equinox, but on the same side, (*viz.* both North or both South of it,) then say, as $R : Co-S$ their $X\text{ }Ing : Co-l.$ Greater $Lt : l.$ a 4th; which taken from the Compl. Lesser Lt . leaves a 5th. Then, as $S. 4th : S. 5th :: Co-S. Greater Lt : S. Distance$ in Degrees, which turn'd into Miles (by Prob. 1.) is the Answer.

3. If the two Places lye, one in $N. Lt$. and the other in $S. Lt$. Then say, as $R : Co-S$, of their $X\text{ }Ing : l.$ Greater $Lt : l.$ a 4th; which taken from the Compl. Lesser Lt , the Remainder will be a 5th. Then say, as $Co-S. 4th : Co-S. 5th :: S. Greater Lt : Co-S. Distance$ in Degrees, which turn'd into Miles (by Prob. 1.) gives the Answer, requir'd.

These are all the most usual Problems in Geography, except what relate to finding the Longitude, Latitude and

and bearing of Places; the two former of which, as they are found by Calculation, they belong as properly to Navigation; to which therefore (together with the Bearing of Places) I shall refer them: Concluding this Chapter with

A brief Explication of the Symbols, made use of (for brevity sake) in the foregoing Problems.

<i>Lng.</i> Longitude.	<i>S.</i> South.
<i>Lat.</i> Latitude.	<i>N.</i> North.
<i>Dst.</i> Distance.	<i>Compl.</i> Complement.
<i>X.</i> Difference.	<i>XLng.</i> Difference of Longitude.
<i>Z.</i> Sum.	<i>XLt.</i> Difference of Latitude.
<i>□.</i> Greater.	
<i>□.</i> Lesser.	

C H A P. VIII.

Of Navigation.

THe principal and most usual Problems in Navigation are such as relate to finding the Longitude, Latitude, Rumb, and Distance of places; any two of which being known, the other two may be found, by the Rules in the following

Problems in Navigation.

Prob.	knowing	to find	The Rule.
1	Lng. & Lt.	Rmb.	<i>As X. Lt : X. Lng :: R : t. Rmb.</i>
2	of 2 Pls.	Dst.	<i>As S. Rmb : R :: X. Lng : Dst.</i>
3	Lng. 2 Pls. and Lt. of	Lt. of other.	<i>As S. Rmb : Co-S. Rmb :: X. Lng : X. Lt.</i>
4	1 of them and Rmb.	Dst.	<i>As S. Rmb : R :: X. Lng : Dst.</i>

Prob.	Knowing	to find	The Rule.
5	Lng. two Pls. & Lt.	Rmb.	$As\ Dst : X. Lng :: R : S. Rmb.$
6	one them, & Dst.	Lt. of the other.	Find Rmb. by Prob. 5. Then say, $As\ S. Rmb : Co-S. Rmb ::$ $X. Lng : X. Lt.$
7	Lt. 2 Pls. and Lng.	Lng. of the other.	$As\ Co-S. Rmb : S. Rmb :: X.$ $Lt : X. Lng.$
8	1 of 'em and Rmb.	Dst.	$As\ Co-S. Rmb : R :: X. Lt : Dst.$
9	Lt. 2 Pl. and Lng.	Rmb.	$As\ X. Lt : Dst :: R : Sec. Rmb.$
10	1 of them and Dst.	Lng. of the other.	Find Rmb. by Prob. 9. Then, $As\ Co-S. Rmb : S. Rmb :: X. Lt :$ $X. Lng.$
11	Rmb. & Dst. two Pls. and	Lng. of the other.	$As\ R : S : Rmb :: Dst : X. Lng.$
12	Lng. or Lt. of one of them.	Lt. of the other.	$As\ R : Co-S. Rmb :: Dst : X. Lt.$

The use of the foregoing Problems, will be made evident by the following Notes.

Note 1. That the Symbols made use of in these Problems, are these that follow.

Ing. Longitude.

Lt. Latitude.

Rmb. Rumb.

Dst. Distance.

Pls. Places.

X. Ing. Difference of Longitude.

X. Lt. Difference of Latitude.

Note 2. If the $\left\{ \begin{smallmatrix} Ing. \\ Lt. \end{smallmatrix} \right\}$ of two *Pls.* be given; Subtract the lesser from the greater, the remainder is the $\left\{ \begin{smallmatrix} X. Lng. \\ X. Lt. \end{smallmatrix} \right\}$ of those two Places.

Note.

Note 3. The *X. Lng.* or *X. Lt.* being thus found, in Deg. reduce the *X. Lt.* into Miles by Chap. 7 Prob. 1. of this Book. Also, reduce the *X. Lng.* into Miles, by Chap. 7. Prob. 2 of this Book. But, in reducing the *X. Lng.* into Miles, observe to do it for *that* Parallel of *Lt.* (not from which, but) *towards* which you are Sailing.

Note 4. If the Rumb be given (as commonly it is) in Points of the Compass; it must be reduced into Deg. by allowing 11 Degrees 15 Minutes for each Point of the Compass.

Note 5. The Rumb is always accounted from the Meridian, towards the East or West, whether you sail towards the North or South.

Note 6. The five precedent Notes being known; any of the 12 Problems, in the foregoing Table, are solv'd, by the Analogy in the fourth Column.

Note 7. The *X. Lng.* or *X. Lt.* found by any of the Problems in this Table, is found in Miles; and must be reduced into Degrees, before you can find the *Lng.* or *Lt.* of the place sought.

Note 8. The *X.* of $\left\{ \begin{array}{l} Lng. \\ Lt. \end{array} \right\}$ in Miles may be reduced into Degrees, by Prob. $\left\{ \begin{array}{l} 13 \\ 14 \end{array} \right\}$ of this Chapter.

Note 9. The *X. Lng.* or *X. Lt.* being thus found, in Degrees; If you seek the *Lng.* of a Place $\left\{ \begin{array}{l} \text{Add} \\ \text{Subtract} \end{array} \right\}$ the *X. Lng.* $\left\{ \begin{array}{l} \text{to} \\ \text{from} \end{array} \right\}$ the *Lng.* of the Place given, if you sail $\left\{ \begin{array}{l} \text{Easterly} \\ \text{Westerly} \end{array} \right\}$ from the Place of known *Lng.* and the $\left\{ \begin{array}{l} \text{Sum} \\ \text{Remainder} \end{array} \right\}$ is the *Lng.* of the Place sought.

In like manner, if you seek the Latitude of a Place,
T 3 Add

{Add } the $X. Lt.$ {to } the $Lt.$ of the Place
 {Subtract } given, if you sail towards the {Pole; } and the
 {Sum } {Equinox; } and the
 {Remainder } is the $Lt.$ of the Place sought.

Note 10. The Rumb being found by any of the Problems of this Table, is found in Degrees, and must be reduced into Points of the Compass, by allowing 11 Deg. 15 Min. for each Point. Then, these Points of the Compass are to be reckon'd from the North or South Points of the Horizon, towards the East or West, and so to be denominat'd respectively, as your own Reason will easily guide you.

Note 11. Because (perhaps) every one (into whose Hands this Book may come) cannot tell the Names of the 32 Points of the Mariners Compass; it may not be much amiss here to insert them, according to their Order; and they are these,

1. North, or full North.
2. North and by East.
3. North North-East.
4. North-East and by ~~East~~ North.
5. North-East.
6. North-East and by East.
7. East North-East.
8. East and by North.
9. East or full East.
10. East and by South.
11. East South-East.
12. South-East, & by East.
13. South-East.
14. South-East & by South.
15. South South-East.
16. South and by East.
17. South, or full South.
18. South and by West.
19. South South-West.
20. South-

20. South-West and by South.

21. South-West.

22. South-West & by West.

23. West South-West.

24. West and by South.

25. West or full West.

26. West and by North.

27. West South-West.

28. North-West & by West.

29. North-West.

30. North-West & by North.

31. North North-West.

32. North and by West.

And so, the next Point comes round to the North again.

Other Problems useful in Navigation, are these
that follow.

P R O B. 13.

To reduce Miles into Great Circle Degrees, (as of the Equinox, or Meridian, &c.)

The R U L E,

As 70 : 1 :: the given Number Miles : the Degrees answering thereto.

P R O B. 14.

To reduce Miles into Degrees of any Parallel of Latitude.

The R U L E

Reduce the given Miles into great Circle Degrees, by Prob. 13. Then say, as Cos. 1st of the Parallel : R : the Number of Great-Circle-Degrees : the Number of Degrees in that Parallel, answering thereto.

P R O B. 15.

To find how many Miles answer to one Degree of Latitude in any Parallel.

The

The R U L E.

As $\cos S.$ Rmb. from the Meridian: $R : : 70 : \text{the Number of Miles answering to one Degree in that Rumb,}$

P R O B. $\left\{ \begin{array}{l} 16. \\ 17. \end{array} \right.$

To reduce $\left\{ \begin{array}{l} \text{Miles} \\ \text{Leagues} \end{array} \right\}$ into $\left\{ \begin{array}{l} \text{Leagues.} \\ \text{Miles.} \end{array} \right.$

The R U L E.

As $\left\{ \begin{array}{l} 3 : 1 :: \\ 1 : 3 :: \end{array} \right\}$ the given Number of $\left\{ \begin{array}{l} \text{Miles :} \\ \text{Leagues :} \end{array} \right\}$
to the Number of $\left\{ \begin{array}{l} \text{Leagues} \\ \text{Miles} \end{array} \right\}$ answering thereto.

C H A P. IX.

Of Dialing.

THe Principal Problems in Dialing, are such as relate to find the Hour Distances, and other Requisites of a Sun-Dial; the Method of performing which, I shall briefly, yet plainly shew in this Chapter. But before I shew how to Calculate the Hour-Distances, it will be necessary (because it must be first known) that I shew how ———

To find the Requisites belonging to any Sun-Dial, &c.

§. 1. For an Horizontal-Dial.

In this there is nothing requir'd but the *Height of the Stile above the Plain*; and that is always equal to the Latitude of the Place.

§. 2. Direct Vertical North or South Plains.

In these Plains also, there is nothing requir'd but the *height of the Stile above the Plain*; and that is always equal to the Compl. of the Latitude of the Place.

§. 4.

§. 3. Vertical Declining Plains.

In these Plains (before the Hour-distances can be Calculated) three things (besides *Lt-Pl.* and *Dcl. Pln.*) must be found, of which in their Order, &c.

Prob. 1. To find the Stile's Height.

Say, as $R : Co-S. Li-Pl :: Co-S. Del-Plm : S. of the$
 Stile's Hight.

Prob. 2. To find the Substile's Dst. from the Meridian.

Say, as R : S. *Dcl-Pln* : : *Co-t. Lt-Pl* : : *t. Dfl.* of the Sub-
stile from the Meridian.

Prob. 3. To find the Plain's X-Lng.

Say, as Co-S. Lt-Pl: R to S. Subtil's Dft. from the Meridian: S. Plain's X-Lng.

S. 4. East or West Reclining or Inclining Plains.

In these Plains (as in Vertical Decliners) three things (besides the Latitude and Reclination) must be found; before the Hour-Distances can be Calculated; and they are the same as in the other, of which in their Order. And

Prob. 1. To find the Stiles Hight.

Say, as R: S. Lt. Pl :: S. Reclination: S. Stile's Height
above the Plain.

Prob. 2. To find the Substyle's Dst. from the Meridian.

Say as, R : Co-S. Reclination :: t. Lt-Pl : t. Subtile
Dlt. from the Meridian.

Prob. 3. To find the Platn's XEng.

Say, as $S. Lt - Pl :: R : S$. Substile's Dft. from the Meridian : $S. Plain's X. Lng.$

S. 5. South Declining { Reclining } Plains.

In these Plains besides the *Lt. Pl.* the Dcl. and Re-
clination

clination, or Inclination of the Plain, four things must be found, before the Hours can be drawn, of which in order. And,

Prob. 1. *To find the Dst. of the Meridian and Horizon.*

Say, as $R : S$. Reclination or Inclination :: t . $Dcl : t$.
Dst. of the Meridian and Horizon.

Prob. 2. *To find the Stile's Hight.*

Say, as $R : S$. *Dst. Meridian and Horizon :: Co-S. Reclination, or Inclination : S. a 4th Arch; which Subtracted from the Lt-Pl. leaves a 5th Arch. Then say, as S. Dst. Meridian and Horizon : S. 5th Arch :: Co-S. Dcl-Pln : S. Stile's Hight.*

Prob. 3. *To find Dst. Substile from the Meridian.*

Say, as $Co-t. Dcl-Pln : t$. Stile's Hight :: that 4th S . (in the preceding Prob.) : S . Substile's Dst. from the Meridian.

Prob. 4. *To find the Plain's XEng.*

Say, as the 5th S . (found in the foregoing Prob. 2.)
 $R :: S$. Substile's Dst. from the Meridian : S . Plain's XEng.

§. 6. North Declining $\left\{ \begin{array}{l} \text{Reclining} \\ \text{Inclining} \end{array} \right\}$ Plain's.

In these Plains (as in South Decliners Reclining) the same four things must be found, before the Hour Distances can be obtain'd, of which in Order. And

Prob. 1. *To find the Dst. of the Meridian and Horizon.*

Say, as $R : S$: Reclination or Inclination :: t . $Dcl-Pln : Co-t$.
Dst. Meridian and Horizon.

Prob. 2. *To find the Stile's Hight.*

Say, as S . $Dcl-Pln : R :: Co-S$. *Dst. Meridian and Horizon : S. a 4th Arch, to which add the Compl. Lt-Pl. then the Compl. of that Sum to 180 Deg. shall be a 5th Arch,*

Arch. Then say, as *S.* 4th Arch : *S.* 5th :: *S.* Reclination : *S.* Stile's Hight.

Prob. 3. To find *Dst.* Substile from the Meridian.

Say, as *t.* Reclination : *Co-S.* *Dst.* Meridian and Horizon :: *t.* Stile's Hight : *S.* a 4th Arch. Which Arch, or its Compl. to 180. Deg. is the Substile's *Dst.* from the Meridian, according as you count it from the North or South.

Prob. 4. To find the Plain's *XLng.*

Say, as *S.* Stile's Hight : *R.* :: *t.* *Dst.* Substile and Meridian : *Co-t.* a 4th Arch. Which Arch is the Plain's *XLng.* from the North; or its Compl. to 180 Deg. if you account it from the South.

S. 7. Direct South and North Reclining Plains.

In these Plains there is nothing requir'd, before the Hour-Distances be Calculated, but the Stile's Hight ; which may easily be found thus —

1. For South Recliners.

If the Reclination be $\left\{ \begin{array}{l} \text{less} \\ \text{more} \end{array} \right\}$ than the Compl. *Lt-Pl.* then the $\left\{ \begin{array}{l} X. \\ Z. \end{array} \right\}$ of the Reclination and Compl. *Lt-Pl.* is the Stile's Hight.

2. For the North Recliners.

Add the Reclination to the Compl. *Lt-Pl.* and their Sum, (if less than 90 Deg.) or its Complement to 180 Deg. (if more than 90 Deg.) is the Stile's Hight.

S. 8. To Calculate the Hour-Distances upon all sorts of Plains.

Upon all Horizontal, and upon all Direct North and South Plains, whether Erect, or Reclining, the Stile stands upon the 12 a Clock Line; for these Plains have no *XLng.* and therefore the Substile and Hour-Line of

are the same, and the Equinoctial Distances of each Hour from 12, is 15 Degrees.

In all other Plains (as Erect Decliners, and North and South Recliners declining,) which have *X^{ing}*. the Plain's *X^{ing}*. must be reduc'd into time, by allowing 15 Deg. for one Hour, i. e. one Deg. for 4 Min. of Time; and so having found the Equinoctial Distances of the Hours on either side of the Substile; the Equinoctial Distances for all the other Hours, are easily found, by the continual Addition of 15 Deg. for an Hour, and then the Proportion will be

As *R* : *S*. Stiles Height above any Plain whatsoever it be : *t*. of the Equinoctial Dst. of any Hours from the Meridian of any direct North or South Plain, whether Erect or Reclining : — but from the Substile of all other Plains : to the *t*. of the true Hour-Dst. upon the Plain, from the Meridian, or Substile.

I shall conclude this third Book, with

A brief Explanation of the Symbols made use of (for brevity sake) in this Chapter.

<i>R</i> . Radius.	<i>i. e.</i> That is
<i>S</i> . Sine.	<i>Compl.</i> Complement.
<i>t</i> . Tangent.	<i>Decl.</i> Declination.
<i>Co-S</i> . Co-Sine.	<i>Dst.</i> Distance.
<i>Co-t</i> . Co-Tangent.	<i>Lt - Pl.</i> Latitude of a
<i>X</i> . Difference.	Place.
<i>Z</i> . Sum.	<i>Del-Pln.</i> Declination of the
<i>X-Lug.</i> Difference of Longitude.	Plain.

BOOK IV.

Of the Use of the Line of Numbers, in Chronology, Dioptricks, Gunnery, Architecture
 { *Military, or Fortification,* } *Mechanick-*
 { *Civil,* }
 { *Nautical,* }
Powers, Automaticks or Clock-Work, and Philosophy.

P R E M E.

IT cannot be expected, That I should here teach the Whole of each Science here mention'd; because each of them (when largely explain'd) is capable to fill a Book as big as this is intended: wherefore my Business here, shall be only to shew how to solve all the most usual and most useful Propositions in each, by the Line of Numbers: Which I shall endeavour to do in 9 Chapters. Whereof —

Chap. 1. Shall treat of Chronology: Shewing how to find the Golden-Number, Epact, Dominical Letter, Cycle of the Sun, Roman Indiction, Number of Direction, Moveable Feasts, New and Full Moon in every Month, &c. for any Year past, present or to come; as also, whether any Year be Leap-Year, or what Year past; what Day of the Week begins the Year; the age of the Moon for any Day of any Month, &c.

Chap. 2. Of Dioptricks; shewing how to find the absolute and Respective Focus's of Convex, Concave

and Meniscus Glasses; either single or combin'd; as also, to find how much they Magnify, either single or combin'd in Telescopes, or Microscopes, &c.

Chap. 3. Of Gunnery, shewing how to find the Weight of several pieces of Ordnance: the Quantity of Powder to charge them; the Weight of their Bullets, &c. the distance they will carry, Point blank, or at any Degree of Mounture, &c.

Chap. 4. Of Military-Architecture or Fortification; shewing how to make any Regular Fortification, from the Tetragon to the Dodecagon.

Chap. 5. Of civil Architecture; shewing how to find the length of Timber in a Building, &c.

Chap. 6. Of Nautical Architecture; shewing how to find the length of Timber in a ship; with its Burden, &c.

Chap. 7. Of the Mechanick Powers; shewing how to find the Force or Power of any Leaver, Wheel, Pulley, Wedge, Skrew, or Inclining plane, or of any Machine compos'd of any two or more of these Mechanick Powers, &c.

Chap. 8. Of Automaticks or Clock Work; shewing how to find the sizes of all Wheels, or Pinions, according to any Number of Teeth, or Leaves assign'd to the Wheels, or Pinions. Also to find the length of Pendulums, to make any Number of Beats, &c.

Chap. 9. Of Philosophy; shewing how to find the strength of any piece of Timber; the time of the Descent of falling Bodies: the time of the Flight of Sound, &c.

C H A P. I.

Of Chronology.

P R O B. 1.

To find the Golden-Number, for any Year.

The R U L E.

TO the Year of our Lord (propos'd) add one, and divide the sum by 19. the Remainder (not regarding the Quotient) is the Golden Number for that Year. If nought remain, the Golden Number is 19. So for the Year 1708. the Golden Number is 18.

P R O B. 2.

To find the Cycle of the Sun, for any Year.

R U L E.

To the Year of our Lord (propos'd) add 9, and divide the Sum by 28, the remainder (not regarding the Quotient) is the Cycle of the Sun. If nought remain, the Circle of the Sun is 28.

Note 1. That the Golden-Number, and Cycle of the Sun, are the same both in the English and Foreign Account.

2. That they always begin the First of January.

P R O B. 3.

To find the Epact, for any Year, both in the English and Foreign Account.

First, for the English Account.

First find the Golden Number (by Prob. 1.) for the Year propos'd; then multiply the Golden-Number by 11, and divide the Product by 30, and the Remainder (not regarding the Quotient) is the Epact, according to the English Account.

Or more easily thus.

Divide the Golden-Number (for the Year propos'd)
by 3, and if $\left. \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \right\}$ remain, the Epact is
 $\left. \begin{matrix} \text{the same as} \\ 10 \text{ more than} \\ 20 \text{ more than} \end{matrix} \right\}$ the Golden-Number; but if it
exceed 30, what remains is the Epact.

Secondly, for the Foreign Account.

Having found the Epact according to the English Account; subtract 11 therefrom, (adding 30 if Subtraction cannot otherwise be made) and what remains is the Epact according to the Foreign Account. If 0 remain, the Epact is 0. If the English Epact be less than 11, add 30 to it, and then Subtract as before.

Note 1. The Epact always begins on the first of *March*.
2. Mr. *Wing* (in his *Ephemerides*) is under a grand Mistake; (either out of *Ignorance*, or rather *Carelessness*, for I think such a Man must needs know better;) for, ever since the Year 1700, he always makes the Epact, in the Foreign Account, one more than it should be, e. g. the Year 1705, he makes the Epact in the English Account 15 (which is right) & in the Foreign Account 5, which should be but 4. Whence he Calculates all the Moveable Feasts (in the Foreign Account) false; and what is worse, (and indeed ridiculous,) he makes all those Moveable Feasts (in the Foreign Account) which should fall on the Sunday, as *Shrove-Sunday*, *Easter-Day*, *Rogation-Sunday*, *White-Sunday*, *Advent-Sunday*, &c.) to fall on the Saturday. So that here the Old Proverb is Verify'd, *Ad Græcas Calendæ, two Sundays come together*, to Day in *France*, &c. to Morrow in *England*. This is much as is reported of Parson *Zachary*, sometime Minister of *Dalington* in *Sussex*; next Year (says he) being Leap-Year, *Easter-Day* will fall upon the *Monday*.

I have not inserted *this* out of any Envy to the Author,

thor, (for the Gentleman is unknown to me;) but to let the World see, how easy it is for us to be impos'd upon, in things we understand not. Nevertheless, if he only had been Guilty of this Error, I had held my Peace, since *Humanum est Errare*: But I may add, that most of our Annual Writers (I fear out of Ignorance) have (to my Knowledge) for several Years, been guilty of the same Error, of making those Days above mention'd, to fall on the *Saturday*, and well they may do so, for I find they transcribe from him *Ala volée*, his very Errors, but I shall leave this tedious Digression, and proceed to

P R O B. 4.

To find the Roman Indiction for any Year

The R U L E.

To the Year of our Lord (propos'd) add 3, Divide the Sum by 15, the remainder (not regarding the Quotient) is the Roman Indiction: This (though commonly inserted in Almanacks) is now of no use, I think I am sure to us; and therefore I shall say no more of it.

P R O B. 5.

To find whether any Year be Leap-Year, or what Year past.

The R U L E.

Divide the Year of our Lord (propos'd) by 4, and the Remainder (not regarding the Quotient) will shew what you want to know. For if nothing remain it is Leap-Year; if 1, 2, or 3, remain, it is the first, second or third Year after Leap-Year. So the Year 1705 was the first after Leap-Year.

P R O B. 6.

To find the Dominical Letter for any Year, both in the English and Foreign Account.

For the English Accompt.

Divide the Year of our Lord (propos'd) by 4, and add the Quotient (not regarding the Remainder) to the Year propos'd, and to their Sum add 4; then divide this last Sum by 7, (not regarding the Quotient, but) Noting the remainder; for the Complement of this Remainder to 7, shall shew the Number of the Letter, reckoning. A 1, B 2, C 3, D 4, &c.

Note. If there be no Remainder when you Divide the Year by 4, that then it is Leap-Year, and consequently there are two Dominical-Letters that Year; whereof this Rule find the *Last*; the *First* is the next Letter in the Order there mention'd, except the last Letter be found to be G, and then the first Letter will be A.

Otherwise,

First find the Cycle of the Sun for the Year propos'd, by Prob. 2, then to the Cycle of the Sun add 3, and divide the Sum by 4; add the Quotient (not regarding the Remainder) to the Cycle of the Sun, and divide this Sum by 7, (not regarding the Quotient, but) Noting the Remainder, for that will shew the Letter, if you count them in this Order, G 1, F 2, E 3, D 4, &c.

Note 1. If when you divide by 7, nothing remain, 7 is supposed to remain, and the Letter is A.

2. If when you divide by 4, nothing remains, it is Leap-Year; and consequently there are two Dominical Letters, of which this Rule finds the *Last*; the *First* is the next in the order of the Alphabet, except the last be G, and then the first is A. *Example*, by either of these ways the Dominical Letter (according to the English Accompt) for the Year 1705, will be found to be G.

For the Foreign Accompt.

Having found the Dominical Letter according to the English Accompt; find the Number of the Letter according to the Order of the Alphabet, (*viz.* A 1, B 2, &c.)

66.) and Subtract three therefrom ; so the Remainder will shew the Dominical Letter in the Foreign Account. Thus for the Year 1704, the Dominical Letter (in the Foreign Account) will be D. But

Note 1. If in Subtraction nothing remain, 7 is suppos'd to remain, and the Dominical Letter is G.

2. That this Rule finds the Dominical Letter (in the Foreign Account) till the Year 1800, for which Year, and so forward till the Year 1900, there must be but one Subtracted. The Reason is, because every 100th Year, for 300 Years together, (but not the 4th) they omit the Leap-Year, viz. in the Year 1700, 1800, and 1900, they omit the Leap-Year ; but not in the Year 2000.

3. The Dominical-Letter (in both Accounts) changes the first Day of January.

4. That when 'tis Leap-Year in one Account, 'tis so also in the other ; except where the Foreigners omit the Leap-Year, as was shewn in the second Note.

5. And when 'tis Leap-Year, there are two Dominical Letters, the first of which is used till St. Matias's Day, (the 25 of Febr.) and the other from thence to the end of the Year.

P R O B. 7.

To find the Number of Direction, for any Year, both in the English and Foreign Account.

Tho' Mr. Wing, Mr. Leybourn, and Mr. Salmon affirm, That this Problem cannot be solved by Arithmetick ; yet I shall shew that it may : but I confess it cost me some time and study to discover the Method of it ; which is thus,

For the English Account.

First find the Epact (by Prob. 3.) for the Year propos'd. Then Subtract the Epact from 26, (adding 30 if Subtraction cannot otherwise be made,) noting the Remainder, which I will call the Fixed Number. Then divide

divide the Epact by 7, (not regarding the Quotient, but) Noting the Remainder, (where Note, if nothing remain, the remainder must be accounted 7,) and take its Complement to 8. This Complement I will call the *Subtrahend* or Number to be Subtracted. Then (having found the Dominical Letter by Prob. 6.) Subtract this *Subtrahend* from the Number of the *Dominical-Letter*, (reckoning, A 1, B 2, C 3, &c.) and the Remainder added to the *Fixed Number*, gives the *Number of Direction*.

Note 1. If the *Subtrahend* be greater than the Number of the Dominical Letter, you must add 7 to the Number of the Dominical Letter) that so Subtraction may be made.

2. That, if (in Subtracting the *Subtrahend* from the Number of the Dominical Letter,) nothing remain, the Remainder must be accounted 7.

3. That when the *Epact* is 28, you must add two to the *Subtrahend*; and when it is 29, you must Subtract 5 from it, and so the Sum, or Remainder will be the true *Subtrahend* for those times.

4. That when it is Leap-Year, there being two Dominical-Letters, you must make use of the last of them.

Thus the Number of Direction for the Year 1705, was found to be 18.

For the Foreign Accompt.

Find the Dominical-Letter in the Foreign Accompt, by Prob. 6, and then work in all respects as in the English Accompt, only making use of the Foreign Dominical-Letter; for the Epact made use of must be the English.

Thus the Number of the Direction for the Year 1705, was found to be 15.

Thus you may find the Number of Direction in either Accompt, without much trouble; Nevertheless it may be found with more ease, by the following

T A.

TABLE.

The Use of this Table is very easy; thus, First find the Prime or Golden-Number (by Prob. 2.) for the Year propos'd. Then find the Dominical Letter

in the { English }
 { Foreign }

Accompt, if you seek the Number of Direction in that Accompt. Then enter this Table with the Prime, in the first Column, and the Dominical Letter at the top, and in the common Angle of meeting you have the Number of Direction.

Example. For the Year 1705, the Prime is 15, and the Dominical Letter G, in the English, and D, in the

Foreign Accompt, what is the Number of Direction in each Accompt? Ans. 18 in the English, and 25 in the Foreign Accompt.

Note. If it be Leap-Year, you must enter the Table with the last of the Dominical Letters, because the first of them is out of Date the 24th of February, and the Number of Direction, begins not till 22 of March.

PROB. 8.

To find the Movable Feasts (and Terms) in any Year, in both Accompts.

1. For

Pr.	A	B	C	D	E	F	G
1	19	20	21	22	16	17	18
2	5	6	7	8	9	10	11
3	26	27	28	29	30	24	25
4	19	13	14	15	16	17	18
5	5	6	7	8	2	3	4
6	26	27	28	29	23	24	25
7	12	13	14	15	16	10	11
8	33	34	35	29	30	31	32
9	19	20	21	22	23	24	18
10	12	13	7	8	9	10	11
11	26	27	28	29	30	31	32
12	19	20	21	15	16	17	18
13	5	6	7	8	9	10	4
14	26	27	28	29	23	24	25
15	12	13	14	15	16	17	18
16	5	6	7	1	2	3	4
17	26	20	21	22	23	24	25
18	12	13	14	15	9	10	11
19	33	34	28	29	30	31	32

r. For the English Accompt.

Find the *Number of Direction* (for the Year propos'd) by Prob. 7. Then add the *Number of Direction*

Jan. 31.				Shrove-Sunday.
Feb. 7.				First Sunday in Lent.
May 21.				Easter-Day.
Apr. 7.				Easter-Term begins.
Apr. 25.				Rogation-Sunday.
to < Apr. 29. >	and you have	<		Ascension-Day.
May 3.				Easter-Term ends.
May 9.				White-Sunday.
May 16.				Trinity-Sunday.
May 21.				Trinity-Term begins.
June 9.				Trinity-Term ends.

2. For the Foreign Accompt.

Find the *Number of Direction* (for the Year propos'd) in the Foreign Accompt, (by Prob. 7,) and add 7 thereto, rejecting 35 if the Sum amount to above 35. Then add this Number to the Months and Days in the preceding Table; so you will have the Moveable-Feasts in the Foreign-Accompt.

Note 1. That this Method gives the Time of the Moveable-Feasts, in the { English } Foreign } Accompt, on such a Day of the { English } Foreign } Month.

2. That *Advent-Sunday* (in both Accompts) is always the nearest Sunday to the Feast of *St. Andrew*, whether it be before it or after it.

P R O B. 9.

To find the Age of the Moon; or the Day of her Full and Change, in any Month of the Year.

Though this cannot be done by the Line of Numbers yet because it shews the Use of the Epact, I shall here shew how it may be done; and for the better remembering the Rule, take it in Verse thus:

Janu,

Janu. 0, 2, 1, 2, 3, 4, 5, 6,
8, 8, 10, 10, *these is the Epact fix,*
The Sum (bate 30) to the Month Day add,
Or take from 30, Age, or Change is had.

Which is thus explain'd. Add to the Epact for *Jan.* 0, *Feb.* 2, *March* 1, &c. When the Epact is added to any of these Numbers, the Sum if it be less than 30, or the Excess above 30, added to the Day of the Month (rejecting 30, if need be) gives the Age of the Moon that Day; but Subtracted from 30, leaves the Day of the Change in (in or from the beginning of) that Month.

Having found the Day of the Change; if it be
 $\left\{ \begin{array}{l} \text{more} \\ \text{less} \end{array} \right\}$ than 15, $\left\{ \begin{array}{l} \text{Subtract 15 from it.} \\ \text{Add 15 to it,} \end{array} \right\}$ so the
 $\left\{ \begin{array}{l} \text{Remainder} \\ \text{Sum} \end{array} \right\}$ shall be the Day of the Full Moon in that Month.

P R O B. 10.

To find what Day of the Week begins the Year: and consequently, what Day of the Week begins any Month in the Year.

Divide the preceding Year by 4, and add the Quotient (not regarding the Remainder) to that Year; which Sum divide by 7, (not regarding the Quotient) and the Remainder is the Day of the Week that begins the Year; reckoning *Sunday* 1, *Monday* 2, &c. If nothing remain, the first Day of *Jan.* is *Saturday*.

To find the first Day of the other Months, take this Distick.

<i>Feb.</i>	<i>Mar.</i>	<i>Apr.</i>	<i>May.</i>	<i>June.</i>
Add 3, and 3,	6,	1,	and 4,	
6,	2,	5,	0,	3, and 5 more.
<i>July.</i>	<i>Aug.</i>	<i>Sept.</i>	<i>Octo.</i>	<i>Nov.</i>
				<i>Dec.</i>

Above and below the Figures in these Verses, you have the Months to which the Figures (in the Verses) refer

as 3 to Feb. 3 to March &c. And the plain meaning of the Rule is, That whatsoever Day of the Week, New-Years Day falls upon; the addition of these respective Numbers, to the Weekly Number of that Day will shew what day of the Week all other Months come in upon, only two Cautions are to be observ'd.

1. If the sum of the Numbers added exceed 7, then 7 is to be abated, and the rest shews the day of the Week.

2. In Leap-Years, you must account *March*, and every Month after, to come in one day of the Week later than will appear by these Rules.

Hence, and by help of the following Table, you may find the day of the Month at any time, as well as by an Almanack.

5	4	3	2	1	The Table.						
29	22	15	8	1	Tu.	Mo.	Tu.	We.	Th.	Fr.	Sa.
30	23	16	9	2	Mo.	Tu.	We.	Th.	Fr.	Sa.	Su.
31	24	17	10	3	Tu.	We.	Th.	Fr.	Sa.	Su.	Mo.
0	25	18	11	4	We.	Th.	Fr.	Sa.	Su.	Mo.	Tu.
0	26	19	12	5	Th.	Fr.	Sa.	Su.	Mo.	Tu.	We.
0	27	20	13	6	Fr.	Sa.	Su.	Mo.	Tu.	We.	Th.
0	28	21	14	7	Sa.	Su.	Mo.	Tu.	We.	Th.	Fr.

The Use of this Table will be plain by the following

Example.

Suppose I would know what day of the Month the third *Saturday* of *March* 1705 was: by the former part of this Prob. I find the first day of the Month was *Thursday*; therefore I find *Thursday* on the top of the Table, and under it (in the same Column) I find *Saturday*, against which on the Left Hand is 3, 10, 17, 24, 31; which shews that they are all *Saturdays*: and 21, being under the Figure 3, at the top of the Table, shews the third *Saturday* to be the 17th day.

On

On the contrary, if you would find what Day of the Week any Day of any Month is, enter the Table on the top, with the Day of the Week that begins the Month; and on the left Side, with the Day of the Month; and at the Angle of meeting is the Day of the Week sought. This needs no Example.

P R O B. 2.

To reduce Jewish or Planetary Hours into our common equal Hours.

The R U L E.

Say, as 12 Hours, is to the length of the { Day } from
Sun { Rising to Sun Setting } ; so is the Jewish or
Planetary Hour propos'd to a 4th. Which added to the
time of the Sun { Rising, } if the Jewish or Planetary
Hour by { Day } be given, the Sum is the Answer.

Example.

When doth the 5th Jewish or Planetary Hour by Day end, when the Days are 16 Hours long? Answ. At 40 Min. after 10 a Clock in the Forenoon. For as $12:16::5:6\frac{2}{3}$ equal to 8 Ho. 40 Min. which added to 4 Ho. (the time of Sun-Rising) gives 10 Hours, 40 Minutes.

C H A P. II.

Of Dioptricks.

P R O B. 1.

To find the Absolute Focus of a Convex-Glass.

The R U L E.

Expose the Glass to the Sun, and you shall see a bright Spot cast upon the opposite Wall; remove the Glass towards or from the Wall, till you bring that bright Spot into the least Compass you can. Then the Distance from the Glass to the Spot being measur'd, shall give you the Absolute or Solar-Focus of that Glass. There are several other ways to find the Absolute Focus; but this being a general Rule for all Convex-Glasses, I shall (for Brevity sake) wave the rest.

P R O B. 2.

To find the Respective Focus of a Convex-Glass.

1. When a nigh Object is placed more distant from the Glass than the Focus; the Rule for determining the Distinct-Base or Respective Focus, is *this*.

As the difference between the Distance of the Object and the Focal length, is the or Focal Length; so is the Distance of the Object from the Glass, to the Distance of the Respective Focus or Distinct Base from the Glass.

2. When a nigh Object is placed nearer to the Glass than the Focus; the Rule for finding the Imaginary Focus from the Glass.

As the difference between the Distance of the Object from the Glass, and the Glass's Focal length, is to the Glass's Focal length; so is the Distance of the Object from the Glass to the Distance of the Imaginary Focus from the Glass.

3. When a nigh Object is placed in the Focus of the Glass; the distinct Base or Respective Focus will be at the same Distance on the other Side of the Glass.

P R O B. 3.

To find the Absolute-Virtual Focus (or Point of Divergency) in Concave Glasses expos'd to Parallel Rays.

The R U L E.

As the Sum of the Radij of both Concavities, is to the Radius of either Concavity; so is the double Radius

radius of the other Concavity, to the distance of the Absolute Virtual Focus from the Glass.

P R O B. 4.

To find the Respective Virtual Focus (or Point of Divergency) in Concave-Glasses, expos'd to converging Rays; when the Point to which the Ray directly Converges, is distant from the Glass, more than the Absolute Virtual Focus in Parallel Rays.

The R U L E.

As the difference between the distance of this Point from the Glass, and the distance of the Absolute Virtual Focus from the Glass, is to the Distance of the Absolute Virtual Focus; so is the distance of this Point of Convergency from the Glass, to the Distance of the Respective Virtual Focus of this Converging Ray.

P R O B. 5.

To find the Respective Focus (or Point where the Refracted Ray crosses the Axis) in concave Glasses, expos'd to converging Rays; when the Point to which the Ray Converges, is higher to the Glass, than the Absolute Virtual Focus of Parallel Rays.

The R U L E.

As the Excess of the Virtual Focal length, above the Distance of this Point of Convergency from the Glass, is to the Virtual Focal length; so is the Distance of this Point of Convergency from the Glass, to the distance of the respective Focus or Point where this Ray crosses the Axis.

P R O B. 6.

To find the Respective Virtual Focus or Point of Divergency, in Concave-Glasses expos'd to Diverging Rays.

The R U L E.

As the Sum of the Distance of the Object from the Glass, and the Glasses absolute Virtual Focal length, is to

X 2

the

absolute Virtual Focal length; so is the Distance of the Object from the Glass, to the Distance of the respective Virtual Focus, (or Point of Divergency) from the Glass.

Note. That by *Focus* or *Virtual Focus*, I mean the absolute Focus, or absolute Virtual Focus, made by Parallel Rays falling in a Convex, or Concave-Glass.

P R O B. 7.

A General Rule, to find the Focus, either Real or Virtual, of any Meniscus Glass; the Semidiameters of whose Convexity and Concavity being unequal.

As the Difference of the Semidiameters, is to either of the Semidiameters, whether of the Convexity or Concavity; so is the Diameter of the other Surface, to the Focal length, Real or Virtual.

Note 1. That if the Semidiameter of the Concave Surface be greater than the Semidiameter of the Convex, the Glass has a real Focus. But, ———

2. If the Semidiameter of the Concave, be less than the Semidiameter of the Convex, the Glass has only a Virtual Focus.

3. That if both the Superficies have the same Diameter, the Ray that falls thereon Parallel to the Axis, after its passing the Glass, proceeds again Parallel.

Thus much of *Single Convex* and *Concave Glasses*. Before I treat of *Compound Glasses*, I shall say something of *Plain Glasses*. Concerning which, Observe

1. That a Ray of Light falling Perpendicularly on a Plain Glass, passes through unrefracted: But all inclined Rays are refracted.

2. That a Ray of light falling inclined on a plain Glass, *Kepler* tells us, That under 30 Degrees of Inclination from Air to Glass, the Angle of Refraction is $\frac{1}{5}$ of the Inclination; and therefore the Refracted Angle is $\frac{2}{5}$ of the Inclination. Wherefore we lay down the following Proportions, as confirm'd by *Kepler's Experiments*, and usually retain'd by most Optick Writers.

P R O B.

P R O B. 8.

To find the Refracted Angle, of a Ray proceeding from Air to (a Plain) Glass.

The R U L E.

As 3 to 2; so is the Angle of Inclination, to the Refracted Angle.

P R O B. 9.

To find the Angle of Refraction, of a Ray proceeding from Air to (a Plain) Glass.

The R U L E.

As 3 to 1, so is the Angle of Inclination, to the Angle of Refraction.

P R O B. 10.

To find the Refracted Angle, of a Ray proceeding from (a Plain) Glass to Air.

The R U L E.

As 2 to 3; so is the Angle of Inclination to the Refracted Angle.

P R O B. 11.

To find the Angle of Refraction, of a Ray proceeding from (a Plain) Glass to Air.

The R U L E.

As 1 to 3; so is the Angle of Inclination, to the Angle of Refraction.

But the Learned Sir Isaac Newton discovered by most accurate Experiments, that these Proportions of Kepler (laid down in Prob. 8, 9, 10, and 11,) were not sufficiently exact: For Des Cartes first found, That Refractions were not to be measured by the Proportion of Angles, but by the Proportion of Sines; (See his Dioptricks, Cap. 2, Sect. 7.) and therefore Sir Isaac Newton apply'd himself to discover the Proportion of the Sines, and lays down these following

P R O B. 12.

To solve the eighth Problem more exactly.

The R U L E.

As 3000 to 193 (or nearly, as 14 to 9;) so is the Sine of the Angle of Inclination, to the Sine of the Refracted Angle.

P R O B. 13.

To solve the 9th Problem more exactly.

The R U L E.

As 300 to 193; so is the Sine of the Angle of Inclination, to the Sine of the Angle of Refraction.

P R O B. 14.

To solve the tenth Problem more exactly.

The R U L E.

As 193 to 300; (or nearly, as 9 to 14;) so is the Sine of the Angle of Inclination, to the Sine of the Refracted Angle.

P R O B. 15.

To solve the eleventh Problem more exactly.

The R U L E.

As 107 to 300; so is the Sine of the Angle of Inclination, to the Sine of Angle of Refraction.

I come now to treat of Compound Glasses.

P R O B. 16.

The Focal lengths of 2 Convex-Glasses being given, the longer being placed before the shorter, at any given Distance less than the longer; To find the Distance of the Compound Focus (or Distinct Base) from the shorter Glass.

The R U L E.

As the Focal length of the shorter Glass, added to the
Diffe.

Difference between the Focal length of the longer and the Glas's Distance, is to the Difference between the Focal length of the longer, and the Glasses Distance; so is the Focal length of the shorter Glas, to the Distance of the Compound Focus (or distinct Base) from the shorter Glas.

P R O B. 17.

A Convex-Glas being given, with a Concave of a larger Sphere, the Concave being placed behind the Convex, at any Distance less than the Focal length of the Convex: To find the place of the Compound Focus (or distinct Base) of these two Glasses.

The R U L E.

From the Focal length of the Convex, subtract the Glasses Distance, mark the difference. Then say, as the Excess of the Focal length of the Concave above this difference is to the Focal length of the Concave; so is this Difference, to the Distance of the Compound Focus (or distinct Base) from the Concave.

P R O B. 18.

The Focus's of a Convex and of a Concave Glas being given, and the Concave placed towards the Object, distant from the Convex, so that the Sum of the Concave's Focal length and Glas's Distance may be greater than the Focal length of the Convex, and this Distance of the Glasses being given also, to find the Distance of the Compound Focus (or distinct Base) from the Concave.

The R U L E.

From the Sum of the Concave's Focal length and Distance of the Glasses, subtract the Focal length of the Convex, mark the Difference. Then say,

As this Difference, is to the Focal length of the Convex; so is the Sum of the Concave's Focal length and Glas's Distance, to the Distance of the Compound Focus from the Convex.

P R O B. 19.

To find how many times the apparent Diameter of an Object is magnify'd by a Telescope, consisting of a Convex Object Glas, and a Convex Eye-Glas.

1 be

The R U L E.

Divide the Focal length of the Object-Glass by the Focal length of the Eye-Glass, the Quotient is the Answer. For, as the Focal length of the Object-Glass, is to the Focal length of the Eye-Glass; so is the apparent Diameter of an Object to the bare Eye, to its apparent Diameter through such a Telescope.

Example.

A Telescope consisting of a 12 Feet Object-Glass, and a 3 Inch Eye-Glass, will magnify 48 times, or make the Object appear 48 times bigger than it does to the bare Eye.

P R O B. 20.

To find the Angle receiv'd by a Telescope of the foregoing Combination.

The R U L E.

As the Distance between the Object-Glass and the Eye-Glass, is to half the breadth of the Eye-Glass; so is Radius, to the Tangent of half the Angle receiv'd.

P R O B. 21.

To find the Diameter of the Image of an Object projected in the distinct Base of a Convex-Glass.

The R U L E.

First find the respective Focus of the Glass, by Prob. 2. Then say, as the Distance of the Object from the Glass, is to the Distance of the Image from the Glass, (which Image is always in the respective Focus;) so is the Diameter of the Object, to the Diameter of the Image.

P R O B. 22.

To find how many times the apparent Diameter of an Object is magnify'd by a single Microscope.

The R U L E.

Divide 8 Inches, by the Focal length of the Glass, the Quotient is the Answer.

P R O B. 23.

To find how many times the apparent Diameter of an Object is magnify'd by a single Microscope.

The

The R U L E.

First find how many times the Object is magnify'd in the distinct Base, by Prob. 21. Then find (by Prob. 22.) how many times of the Image is magnify'd by the Eye-Glass; Multiply one Magnification by the other, the Product is the Answer.

C H A P. III.

Of Gunnery.

P R O B. 1.

The Diameter and Weight of one piece of Ordnance being given; To find the Weight of another Piece of Ordnance (of the same Metal and Shape,) Whose Diameter is also given.

The R U L E.

Say, by Triplicate Proportion, (See Book I. Chap. 16.) As the Diameter of the Piece whose weight is known, is to the Diameter of the Piece whose weight is sought; so is the Weight known, to the weight sought.

P R O B. 2.

Knowing the Diameter and weight of a Brass, or Iron, Piece of Ordnance; to find the Weight of an Iron, or Brass, Piece of the same shape, whose Diameter is also given.

The R U L E.

First find the Weight (by Prob. 1.) as if it were the same Metal with that whose weight is given. Then, if the weight of a Brass, or Iron, Piece be sought; say, As $\left\{ \begin{array}{l} 16:18:: \\ 18:16:: \end{array} \right\}$ so is the Weight found, to the Weight sought.

P R O B. 3.

To find the Quantity of Powder for the Charge of any well Fortify'd Gun, either Brass or Iron, the Weight of the Gun being given.

Note L. That a Gun is said to be well Fortify'd, when the Thickness of the Metal at the Touch-hole, is equal to the Diameter of the Bore.

2. That

2. That in Iron, or Brass Guns, Gunners allow 3, or 4, Ounces of Powder to every Hundred Weight of Metal in the Gun. This being Premis'd, this is

The R U L E.

Multiply the weight of the Gun by 1 for Iron, and by 4, for Brass Guns the Product is the Ounces of Powder for the Charge, which divided by 16 brings it into Pounds.

P R O B. 4.

To find the Quantity of Powder, for the Charge of a Gun not truly Bor'd.

The R U L E.

First, find (by Prob. 3.) the Quantity of Powder for a Charge, supposing the Gun to be truly bor'd.

Secondly, Take the Diameter of the Metal of the Piece at the Touch-hole call this the greater Diameter. Also, add the Diameter of the Bore, to double the thickness of the Metal at the thinnest side, call this the lesser Diameter.

Thirdly, Say, (by Triplicate Proportion) as the Greater Diameter, is to the lesser Diameter; so is the Quantity of Powder (before found) for the true Bore, to the Quantity to be allow'd to the false Bore.

P R O B. 5.

Knowing the Diameter of the Bore of one Gun, and its Charge of Powder: To find the Quantity of Powder, for the Charge of any other Gun, the Diameter of whose Bore is also given.

The R U L E.

1. Say (by Triplicate Proportion) as the Diameter of the Bore of the Gun whose Charge is known, is to the Diameter of the Bore of the Gun, whose Charge is sought; so is the Charge of Powder known, to the Charge sought. But, Note, This Rule supposes the same Proportion betwixt the Weight of the two Guns, as betwixt the Diameter of their Bores. And therefore,

2. To

2. To find whither they have the same Proportion betwixt their Weights, as betwixt the Diameter of their Bores; Work by Prob. 1. thus, — Say, (by Triplicate-Proportion) as the Diameter of the Bore of the Piece, whose Charge is given, is to the Diameter of the Bore of the Piece, whose Charge is sought; so is the Weight of the former to the Weight of the later. If the Weight of the Gun whose Charge is sought, be the same as is found by this Proportion; then the Analogy in the first Section of this Problem, finds the Charge of Powder sought: But if the Weight be not the same as is found by this Proportion; then, to find its due Charge, say, as the Weight of the Gun, found by this Proportion, is to its true Weight; so is the Charge of Powder, found by the Analogy in the first Section of this Problem, to the true Charge of Powder sought.

3. And if there be a stronger Powder than the former, by such Proportion (suppose) as 5 to 2, then to find how much of this sort will charge a Gun: say, as 5 to 2; so is the Charge in the first sort, to the Charge in the strongest sort.

P R O B. 6.

To find how much of every kind of Metal is contain'd in any Brass Piece of Ordnance.

*Tis a usual thing for Gun-Founders, for every 100 Pound of Copper, to put in 10 Pound of Latten, and 8 Pound of pure Tin. Now, suppose this to be the constant Rule for their Mixture, then say, as 118, (the

Sum of the Parts, 100, 10 and 8) is to $\left. \begin{array}{l} 100; \\ 10; \\ 8; \end{array} \right\}$

so is the whole Weight of the Gun, to the Pounds of

$\left\{ \begin{array}{l} \text{Copper} \\ \text{Latten} \\ \text{Tin} \end{array} \right\}$ contain'd therein.

P R O B.

P R O B. 7.

To find how much of each Ingredient, is contain'd in any Quantity of Powder.

Powder, in this Age, is commonly made of 12 Parts of Nitre, 3 Parts of Coal, and 2 Parts of Sulphur. Now, supposing this the constant Proportion for the Mixture; then say, as 17, (the Sum of the Parts 12, 3

& 2) is to $\left\{ \begin{array}{l} 12; \\ 3; \\ 2; \end{array} \right\}$ so is the whole Weight (in Pounds)

of Powder propos'd, to the Pounds of $\left\{ \begin{array}{l} \text{Nitre} \\ \text{Coal} \\ \text{Sulphur} \end{array} \right\}$ contain'd therein.

P R O B. 8.

Knowing the Weight (in Pounds) of an Iron Ball, fit for any Piece of Ordnance, the Periphery of whose Breech (in Calibers) is also given; To find the Charge of Powder fit for Proof and Service.

The R U L E.

As 1, is to the Weight of the Ball in Pounds; so is the Periphery of the Breech in Calibers, to a fourth. Again, as 96 (a fix'd Number,) is to 8, or 6, (other fix'd Numbers) so is the fourth (last found) to the Pounds of Powder for Proof, or Service.

P R O B. 9.

To find the Weight of any Iron Bullet, its Diameter being given.

The R U L E.

Say, (by Triplicate Proportion) as 4, is to the Diameter in Inches; so is 9, to the Weight in Pounds.

Otherwise,

Say, as 100, is to the Cube of the Diameter in Inches; so is 14, to the Weight in Pounds.

P R O B.

P R O B. 10.

To find the Weight of a Granado Shell.

The R U L E.

First find its Weight (by Prob. 9.) as if it were Solid. Then put in a Stick at the Fuse-hole, and take the Diameter of its Concavity, and find its Weight, (as before) as if it were Solid. Subtract this Weight from the former; so the Remainder shall be the Weight of the Granado Shell sought.

P R O B. 11.

To find what Quantity of Powder will fill any Granado-Shell, or Cartridge.

It has been often experimented in the Tower of London, That one Pound of Powder will fill 31.06 Cubical Inches. This being known, the Proportion will be,

As 31.06, is to 1; so is the Concavity (in Inches) of any Granado-Shell, or Cartridge, to the Pounds of Powder that will fill it.

P R O B. 12.

The Diameter of a Cartridge, and the Pounds of Powder that will load any Piece of Ordnance, being known; To find how high the Cartridge must be fill'd.

The R U L E.

First find (by Book 2. Chap. 3. Prop. 3.) the Area of the Circle of the Cartridge, in Inches. Then say,

As the Area of the Circle in Inches, is to the Pounds of Powder allow'd for loading; so is 31.06, to the Depth (in Inches) the Cartridge must be fill'd.

P R O B. 13.

To find the Horizontal Range of any Piece of Ordnance; its greatest Range and Elevation being given.

The R U L E.

As Radius, is to the Sine of double the Elevation; so is the greatest Range, to the Horizontal Range at that Elevation.

Y

P R O B.

P R O B. 14.

To find the greatest Range of any Piece of Ordnance; its Elevation, and Horizontal Range being given.

The R U L E.

As the Sine of double the given Elevation, is to Radius; so is the Horizontal Range, to the greatest Range.

P R O B. 15.

Knowing the greatest Range, and Horizontal Range, of a Piece; To find its Elevation.

The R U L E.

As the greatest Range, is to the Horizontal Range; so is Radius, to the Sine of double the Elevation.

The following Table being useful in Gunnery; I shall with it, put an end to this Chapter.

Names of several Pieces of Ordnance us'd in England.	Guns Wei. in lb.	Len. of the Chase		Dia. of the Bore		Weight of the Ball		Greatest Range Paces.
		Feet.	Inch.	Inch.		lb.	Pou. der lb.	
Rabinet.	120	3	1.25	.75	.5			3769
Falconet.	210	4	2.	1.25	.9			4398
Falcon.	700	6	2.75	2.25	1.1			4797
Minion.	1000	8	3.	4.	2.1			5864
Saker.	1600	9.5	3.5	5.25	4.			5654
Demi-Culverin.	2600	10.	4.5	9.	7.5			4886
Culverin.	4000	11.	5.	15.	10.			4837
Demi-Canon.	6000	11.	6.	27.	14.6			4031
Whole Canon.	7000	12.	7.	47.	21.5			3769
Canon Royal.	8000	12.	8.	63.	27.14			3298
Culverin at Deal Castle. }	—	16.	4.75	—	13.1			7407
Basilisk at Dover Castle. }	—	23.5	4.6	—	18.			11232

Note. A Musket-Shot (to do Execution) is about 720 Feet or 44 Rods.

CHAP.

C H A P. IV.

Of Military-Architecture or Fortification.

TO keep to my Title, That which in this Chapter, will fall under my cognizance, will be the Solution of this General

P R O P O S I T I O N.

To find all the Lines and Angles, of any Regular Fortification, from the Trigon to the Dodecagon; any one of the Lines only been given.

Before I lay down the Rule for solving this Proposition, it may be convenient to premise these following Particulars, Namely,

1. That this Proposition comprehends no less than 1944, distinct Problems; all solvable by one Plain, Easy, and General Rule hereafter to be laid down.

2. That my Design, all along being Brevity, and I now fearing, the Book will swell beyond the intended Bulk of a Pocket-Volume; I shall (for the easier solving of this Problem) lay down the following Table, as being the approv'd Proportions of the most eminent Engineers.

3. That the Angles in all Regular Fortifications of the same kind, are the same, *e. g.* If the Angles in one Regular Tetragonal Fortification are known, they are also known in any other Regular Tetragonal Fortification, for they are the same. This being Premis'd, I here lay down.

A T A B L E.

Of the Dimensions, of any Regular Fortification, from the Tetragon to the Dodecagon, the Flanking Angle being 15 Degrees more than half the Angle of the Polygon.

The Names of the Angles.	Terra.	Penia.	dext.	Hept.	Utra.	Nonat.	Octat.	Emode.	Dode.
Angle of the Polygon.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
Flanking $\angle$ of the Bulwark.	90.00	108.0	120.0	128.34	135.0	140.0	147.16	150.0	
Angle of the Shoulder.	60.0	69.0	75.00	79.17	82.30	85.0	87.0	88.38	90.0
inward Flanking $\angle$.	105.0	109.30	112.30	114.36	116.15	117.30	118.30	119.19	120.0
Outward Flanking $\angle$.	150	139.30	22.30	24.30	26.15	27.30	28.30	29.19	30.0
$\angle$ Froiling the Flank.	156.0	141.0	135.0	130.43	127.30	125.0	123.0	121.0	120.0
The Names of the Lines.	R. 100	R. 100	R. 100	R. 100	R. 100	R. 100	R. 100	R. 100	R. 100
Curtain.	42.0	42.0	42.0	42.0	42.0	42.0	42.0	42.0	42.0
Front of the Bulwark.	28.0	28.0	28.0	28.0	28.0	28.0	28.0	28.0	28.0
Gorge-Line.	10.7	12.18	13.26	14.12	14.83	15.43	15.92	16.36	16.73
Half Diam. of the inw. Pol.	44.92	56.45	68.12	80.94	93.63	106.49	119.49	132.57	145.80
side of the inward Pol.	63.53	66.36	68.52	70.24	71.66	72.84	73.85	74.71	75.47
Perpend. of Ditto.	31.76	45.67	59.34	72.80	86.40	100.4	113.64	127.20	140.83
Half Diam. of the outw. Pol.	67.95	80.63	93.74	107.5	120.50	134.3	147.59	161.16	174.83
Side of the outward Polygon.	96.9	94.79	93.74	92.90	92.21	91.67	91.21	90.85	90.50
Perpend. of Ditto.	48.4	65.23	81.18	96.42	111.32	125.93	140.37	154.63	168.87
Distance of the Polygons.	15.20	19.57	21.84	23.52	24.83	25.87	26.72	27.43	28.4
Flank.	9.3	10.22	11.13	11.85	12.44	12.94	13.36	13.72	14.4
Lead-Line.	23.2	24.16	22.22	24.11	25.87	27.57	28.10	28.19	29.3
Shoulder from the Centre.	14.5	15.90	17.31	18.4	19.56	20.13	20.79	21.31	21.85
second Flank.	8.25	13.12	15.13	16.14	16.77	17.14	17.39	17.56	17.65
longest Line of Defence.	70.61	71.14	71.30	71.43	71.64	71.67	71.77	71.86	71.94

In the upper Part of the Preceeding Table, the first Column contains the Names of the Angles; and all the following Columns, shew the Quantity of those Angles, in Degrees and Minutes. In the lower Part of the Table, the first Column contains the Names of the Lines, the others shew their length, in Rods and Centesimal Parts.

This

This is sufficient for the Explanation of the Table; only we may here Observe, That since the Angles of all Regular Fortifications (whether greater or less,) of the same kind, are equal; it will follow, that 54 of 1944, are always ready solv'd, by means of the former part of the Table) by the following

General R U L E.

As the Number in the Table, respecting the *Datum* (or thing given,) is to the *Datum*; so is the Number in the Table, respecting the *Quæsitum*, to the *Quæsitum* or thing sought.

Example. Let it be requir'd to make a Regular Tetragonal Fortification, the side of whose inward Polygon shall be 31.765 Rods.

In this Case I say (from the Table,) As 63.53 : 31.765 :: 42 : 21, the length of the Curtain.

Again, as 63.53 : 31.765 :: 28 : 14, the Front of the Bulwark.

Again, as 63.53 : 31.765 :: 10.75 : 5.385 the Gorge-Line.

By working in the same manner, the length of all the Lines in the given Fort, will be found to be, as in the following

T A B L E.	Rods.
The Curtain	21.000
The Front of the Bulwark	14.000
The Gorge-Line	5.385
The $\frac{1}{2}$ Diameter of the inward Polygon	22.460
The Side of the inward Polygon	31.765
The Perpendicular of the inward Polygon	15.882
The $\frac{1}{2}$ Diameter of the outward Polygon	33.975
The side of the outward Polygon	48.045
The Perpendicular of the outward Polygon	24.020
The Distance of the Polygons	8.140
The Flank	4.015
The Head-Line	11.510
The Distance of the Shoulder from the Centre	7.025
The Second Flank	4.145
The longest Line of Defence	35.470

The Angles of this Fortification, are to be taken out of the former Part of the foregoing Table, after this manner.

	D. M.
The Angle of the Polygon	90.00
The Flanking Angle of the Bulwark	60.00
The Angle of the Shoulder	105.00
The Inward Flanking Angle	15.00
The Outward Flanking Angle	156.00
The Angle Fronting the Flank	40.00

Thus all the Lines and Angles of this Fortification are found, So that you see here was 14 Problems solved from that one *Datum*, the side of the inward Polygon; and all relating to the Tetragonal Fortification. There may be a like Number relating to each of the other Polygons; which increases the Number to 125, all solvable from the same *Datum*: and there being 15 Lines, each of which may be the *Datum*, it makes up the Number of Problems solvable by this Rule, 1890.

'Twere needless to give any more Examples of this kind, the thing is it self so very plain and easy. I shall therefore (for this time) here put an end to this Chapter of Fortification, and proceed to

C H A P . V.

Of Civil-Architecture.

BY Civil-Architecture, I would be understood to mean, that kind of Architecture which relates to the building of Houses. And here what is Pertinent to our Purpose, is the Solution of the following Problems.

P R O B. 7.

To find the length of the Timber, &c. of a Square Building, at true Pitch: the breadth of the House being given.

For the easier solving of this Problem, take the following Table.

Na res

Names of the Timbers, &c.	Feet.
The breadth of the House	20.00
The length of the Rafter	15.00
The length of the Hip	18.02
The length of the Perpendicular	11.18
The Half Diagonal	14.14

In this Table, the breadth of the House being 20 Feet; the length of the Timber, &c. are as is there express'd. Whence 'tis easy to find them in any other Square Frame, by this

R U L E.

As 20, is to the breadth (in Feet) of any Square Frame; so is the Number (in the Table) against any one of the Timber, &c. to the respective length of that Timber, &c.

Examp. Let there be given the square Frame of a House, 40 Feet broad; and let the Rafters be design'd to be true Pitch, what must the length of the Rafters, and Hips of this Building.

To answer this Question, I say (by the Rule) as 20 : 40 :: 15 : 30, the length of the Rafters. Again, I say, as 20 : 40 :: 18.02 : 36.04, the length of the Hips. In the same manner, the length of the Perpendicular will be found to be 22.36, and the half Diagonal 28.28. So that you see this Problem in 4 Fold, or comprehends 4 Problems.

P R O B. 2.

To find the length of the Collar Beams, in any Square Frame, at true Pitch.

The R U L E.

As the length of the Rafters (which may be found by Prob. 1.) is to the breadth of the House; so is the Distance from the top of the Rafters to the Collar-Beam, to the length sought.

Examp. Let the Frame be 20 Feet wide, the Rafters 15 Feet long, and the Collar-Beams 6 Feet from the top; what must their length be? Ans. 8 Feet. For, as 15 : 20 :: 6 : 8.

P R O B.

P R O B. 3.

To find the length of the Purloyns, in a Square-Frame, Hipt at one end, or both ends, the Rafter being true Pitch.

The R U L E.

First find the length of the Rafter, by Prob. 1. Then say, as the length of the Rafter, is to the breadth of the House; so is the design'd Distance of the Purloyn from the top of the Rafter, to a 4th. Then, if the House be Hipt but at one end, or at both ends, subtract the $\frac{1}{2}$ or the whole breadth of the House, from the Remainder; to this remainder add the 4th, or double the 4th Proportional found, their Sum shall be the length of the Purloyn sought.

P R O B. 4.

To find the length of the Rafter, in a Square Frame at any Pitch.

Before I give the Rule to solve this Problem, it may be necessary to Note, That the breadth of the House is always given; and the length of the Rafter, or Height of the Perpendicular, is next resolved on; so that if one of them be sought, the other is (always) known by Assumption. This being premis'd, the Prob. may be solv'd by this

R U L E.

To the Square of half the breadth of the House, add the Square of the Perpendicular; their Sum is the Square of the Rafter's length. Therefore, the Square Root being extracted, gives you the length of the Rafter, sought.

P R O B. 5.

The Rafter's length being given; To find the Perpendicular, in any Square Frame.

Sometimes, (as I noted above) the length of the Rafter are assign'd, and then the Perpendicular must be found; which may be done by this

R U L E.

R U L E.

From the Square of the Rafter's length, subtract the Square of half the breadth of the House; the Square Root of the Remainder is the Perpendicular sought.

P R O B. 6.

To find the half Diagonal (or distance from the Corner to the King-Post) in any Square Frame.

The R U L E.

Square the half breadth of the House, which Multiply by two, the Square Root of the Product, is the half Diagonal sought.

P R O B. 7.

To find the length of the Hips, in a Square Frame at any Pitch.

The R U L E.

First find the length of the Rafter, by Prob. 4. Then to the Square of the Rafter's length, add the Square of the half breadth of the House; the Square Root of the Sum is the length of the Hip.

Otherwise,

First find the half Diagonal, by Prob. 6. Then, to the Square of the half Diagonal, add the Square of the Perpendicular; the Square Root of the Sum, is the length of the Hip.

P R O B. 8.

To find the length of the Rafter, in a Bevel Frame, at any Pitch.

The R U L E.

To the square of the half breadth of the House (taken Perpendicularly, and not according to the Beveling,) add the Square of the Perpendicular; the Square Root of the Sum, is the length of the Rafter.

P R O B. 9.

To find the Perpendicular, in a Bevel-Frame, the length of the Rafter being given.

The R U L E.

From the Square of the Rafter's length Subtract the Square of the half breadth of the House, (taken as in Prob.

Prob. 8.) the Square Root of the remainder is the Perpendicular.

P R O B. 10.

To find the half Diagonals, in a Bevel Frame.

In a Bevel Frame, the half Diagonals are unequal; to find them this is

The R U L E.

Consider the Bevelling (*i. e.* how much the Bevelling makes the Frame longer than the Square from the shortest Corner, and take the half thereof. Let this half be added to, or Subtracted from, the half breadth of the Frame, (taken according to the Bevelling) noting the Sum, or Remainder; For, if to the Square of this Sum, or Remainder, you add the Square of half the breadth of the Frame, (taken Perpendicularly) the square Root of the Sum shall give you the longer, or shorter, Diagonal sought.

P R O B. 11.

To find the Length of the Hips, in a Bevel Frame.

The R U L E.

First find the length of the Rafter, by Prob. 8. Then add, or Subtract, half the Bevelling to, or from, the half breadth of the Frame, (taken according to the Bevelling) and to the Square of the Sum, or Remainder, add the Square of the Rafter's length; the Square Root of the Sum, is the longer, or shorter Hip.

Otherwise,

First find the half Diagonal by Prob. 10. Then, to the Square of the Perpendicular, add the Square of the longer or shorter half Diagonal; the Square Root of the Sum is the longer, or shorter Hip.

P R O B. 12.

To find the length of the Rasters, in a Taper Frame, being broader at one end than at the other. In

In this Case the Perpendicular is the same throughout the Roof, but the length of the Rafter are unequal: Therefore, to find the length of each pair of Rafter, the breadth of the House must be taken betwixt their Feet; and then the Operation is the same as in Prob. 4.

Note 1. That to the lengths of Timber found by these Rules, you must add sufficient length, for the Feet of the Rafter, and Tenons of the Collar-Beams, &c.

2. That the length of Sleepers, in all Roofs, are the same as the Hips: A Sleeper being the same in the middle of a Roof, as a Hip at the End.

3. That 'tis difficult to find the Diagonals, and Hips, in Frames Taper and Bevelling, without Sines and Tangents, (for I perceive that Mr. Brown, in his Appendix to *Seamozzi's Architecture*, goes upon wrong Principles,) and therefore I shall at this time wave it.

CHAP. VI.

Of Nautical Architecture.

BY Nautical Architecture, I mean that kind of Architecture, that relates to the Building of Ships. This being premis'd, I proceed to

PROB. I.

By the Mould and Burthen of one Ship known, to build another Ship of the same Mould, of any assign'd Burthen, greater, or less.

The RULE.

Say, by Sub-Triplicate Proportion, (See Book 1. Chap. 16. §. 2.) As the Burthen of the Ship, the lengths of whose Timber are known, is to the Burthen of the Ship, the lengths of whose Timber are sought; so is the length of each particular Timber known, to their length sought.

Examp.

Examp. $\begin{cases} 1, 2, 3, \\ 4, 5, 6. \end{cases}$

Suppose a Ship of 100 Tun, be 44 Feet long by the Keel; 20 Feet broad at the Midship-Beam, and 9 Feet deep in the Hold; and do rack it with the Stem forward 13 Feet, and offward 7 Feet: What must be the length by the Keel, breadth at the Beam, and depth in the Hold; and what the Rack forward, and offward, in a Ship of 200 Tun Burthen.

Answer. In this Case, (working by the Rule above mention'd) the Extent to $\frac{1}{2}$ of the Distance betwixt 100 and 200, will reach from the Dimensions given, to those sought. Thus you will find, that for a Ship of 200 Tuns Burthen, of the same Mould with that proposed.

The length of the Keel, must be 55.44 Feet. *i. e.* 55 Feet, and about $5\frac{1}{2}$ Inches.

The breadth at the Midship Beam, must be 25.2 Feet *i. e.* 25 Feet, $2\frac{1}{2}$ Inches.

The Depth in the Hold, must be 11.34 Feet, *i. e.* 11 Feet and about 4 Inches.

The Rack forward must be 16.38 Feet, *i. e.* 16 Feet, $4\frac{1}{2}$ Inches.

The Rack offwards, must be 8.82 Feet, *i. e.* 8 Feet, $9\frac{1}{2}$ Inches.

'Twere needless to Multiply Examples; those already given being sufficient, to make the Rule intelligible to the meanest Capacity: Besides the Reader will find one Example more of this kind, in Book 1. Chap. 16. §. 2. Example 6.

P R O B. 2.

By the Mould and Burthen of one Ship; To find the Burthen of another Ship of the same Mould.

The R U L E.

Say, by Triplicate proportion (See Book 1 Chap. 16.)
As the length of the Keel, (or breadth at the Beam, or depth

depth in the Hold,) of the Ship whose Burthen is known, is to the length of the Keel, (or, &c.) of the Ship whose Burthen is sought; so is the Burthen of the Ship known, to the Burthen of the Ship sought.

Example.

If a Ship whose Burthen is 300 Tun, be 75 Feet in the Keel; What shall be the Burthen of another Ship, (of the same Mould) whose Keel is 100 Feet? Ans. 713 Tun. For the Extent from 75 to 100 will reach (being tripl'd) from 300 to 713.

P R O B. 3.

To find the length of the Main-Mast.

The R U L E.

As 5, is to 12; so is the Breadth of the Ship at the Beam, to the length of the Main-Mast.

Example. If the breadth of a Ship at the middle Beam, be 30 Feet; What must be the Length of the Main-Mast? Ans. 72 Feet.

P R O B. 4.

To find the length of the Fore-Mast.

The R U L E.

First find the length of the Main-Mast, by Prob. 3. Then say, as 5, is to 4; so is the length of the Main-Mast, to the length of the Fore-Mast.

Example If the breadth of a Ship at the Beam, be 30 Feet; What must be the length of the Fore-Mast? Ans. 57.6 Feet, i. e. 57 Feet, and about 7 Inches.

P R O B. 5.

To find the length of the Mizzen-Mast.

The R U L E.

First find the length of the Main-Mast, by Prob. 3. Then say, as 2, is to 1; so is the length of the Main-Mast, to the length of the Mizzen-Mast.

Z

Example.

Example. If the breadth of a Ship at the Beam, be 30 Feet; What must the length of the Mizzen-Mast? Ans. 36 Feet.

P R O B. 6.

To find the length of the Bolt-Split.

The R U L E.

Captain Smith (in his Sea-Man's Grammar) tells us that the Bolt-Split must always be equal with the Fore-Mast; but surely 'tis a mistake of the Printer; and perhaps he would say, it must equal with the Mizzen-Mast; for every one knows, that 'tis nothing near so long as the Fore-Mast.

P R O B. 7.

To find the bigness of each Mast.

The R U L E.

As 3, is to 1; so is the length of each Mast in Feet to its Diameter or Thickness, in Inches.

Example. If the Main-Mast of a Ship be 72 Feet, long; how big must it be at the lower end? Ans. 24 Inches Diameter.

P R O B. 8.

To find the length of the Main-Yard.

The R U L E.

As 6, is to 5; so is the length of the Keel, to the length of the Main-Yard.

Example. If the Ship be 76 Feet at the Keel; What shall be the length of the Main-Yard? Ans. 63.333 Feet, *i.e.* 63 Feet and 4 Inches.

P R O B. 9.

To find the bigness of the Main Yard.

The R U L E.

As 4, is to 1; so is the length of the Main-Yard, in Feet, to his Diameter or Thickness, in Inches. This needs no Example.

P R O B.

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P R O B. 10.

To find the length of the Fore-Yard.

The R U L E.

First find the length of the Main Yard, by Prob. 8. Then say, as 5, is to 4; so is the length of the Main-Yard, to the length of the Fore-Yard.

Note 1. The Mizzen-Yard is commonly as long as the Mast; the Top-Yards, half as long as the Main, and Fore-Yard; the Top-Gallant-Yards half the length of them. But,

2. 'Tis difficult (if not impossible) to give exact Arithmetical Proportions, for the building of all sorts of Ships, and fitting their Masts, Yards, Cables, Cordage, and Sails; by reason their Lengths, Breadths, Depths, Rakes, and Burthens are so variable. And let me add, That 'tis the Opinion of Seamen, That no Artist can build a Ship, and fit her Masts, &c. so truly to Proportion; but upon the Tryal of her Condition, they may be amended, I shall therefore (at this time) put an end to this Chapter, with the following

P R O B. 11.

To find the Burthen of a Ship, or how many Tuns she will carry by knowing her Length at the Keel, Breadth at the Midship-Beam, and Depth in the Hold.

The R U L E.

Say, (1) As 1, is to the Length of the Keel; so is the Breadth at the Midship-Beam, to a 4th. (2.) As 1, is to this 4th; so is the depth in the Hold, to a 7th. (3.) As 95, (for Merchant Ships) is to 1, so is this 7th, to the Ships Burthen sought. But for ships of War, (where there is an allowance for Guns, Anchors, Cables, Masts, Sails, &c. which are all a Burthen, but no Tunnage; the first Term of the third Analogy must be 100.

This is the common Rule; but not exact; yet it may serve to give a guess at the Tunnage of a Ship; but there is a great deal more requir'd to give true Measure.

Mr. *Hunt* (in his *Mathematical Companion*) tells us, That the way us'd by Ship-Wrights, (as he receiv'd it from Mr. *Castell*, Master-Builder at *Deptford*,) is thus,

(1.) As 1, is to the length of the Keel to the Rake; so is the extreame breadth from Out-side to Outside, to a 4th. (2.) As 1, is to this 4th; so is half the breadth, to a 7th. (3.) As 94, (a fix'd Number) is to 1; so is this 7th, to the Ships Burthen sought.

Note. The Dimensions must be all given in Feet.

C H A P. VII.

Of the Mechanic Powers,

P R O B. 1.

To find the Force of any Leaver.

The R U L E.

As the Length of the shorter end from the Fulciment, is to the length of the longer end from the Fulciment; so is 1, to the Force or Power of that Leaver.

Example. If a Leaver be 10 Feet long, and the Fulciment or Prop, at two Feet from one end, what will be the Force of that Leaver? Answ. 4 Fold. (For, as 2, is to 8; so is 1 to 4,) i.e. one Man by the help of this Leaver thus dispos'd, may raise as great a Weight as 4 Men (or 4 times as much as he himself can raise without it.

P R O B. 2.

To find what Power will be sufficient, to raise a given Weight, with a given Leaver.

The R U L E.

As the distance of the Power from the Fulciment, is to the Distance of the Weight from the Fulciment; so is the Weight, to the Power.

Example. If a Leaver be 11 Feet long, and have a Fulciment at one Foot from one end, and a Weight of a 1000 Pound

Pound resting on the shorter end ; What Power apply'd to the longer end, will be able to raise this Weight? Ans. A Power equivalent to 100 Pound Weight? (For, as 10, is to 1 ; so is 1000 to 100.) Therefore one Man, by the help of this Leaver, may easily raise 1000 Pound Weight.

Note. Tho' I here say, that 100 Pound at the longer end of this Leaver, will raise 1000 Pound at the shorter end ; yet by the strictness of these Mechanic Calculations, as appears by Prob. 1.) it is but equal to it in Weight; and therefore you (perhaps) may think it cannot be sufficient to raise it ; Nevertheless my Assertion holds good, namely, that it will be sufficient to raise it ; which comes to pass, by reason there is always added to the Power of any Leaver, the weight of the longer end, which is (many times) very considerable ; as will be evident from Prob. 4.

P R O B. 3.

To find what Weight may be raised by a given Power, with a given Leaver.

The R U L E.

As the Distance of the Weight from the Fulcrum, is to the Distance of the Power from the Fulcrum ; so is the Power, to the Weight.

Example. Thus, with the same Leaver, and dispos'd as was exemplify'd in Prob. 2. one Man or Power equivalent to 100 Pound, will be able to raise a Weight of 1000 Pound.

P R O B. 4.

To find what Weight must be apply'd to the shorter end (from the Fulcrum) of any Leaver, to make it equiponderate with the longer end. Or, (which is the same thing in effect,) To find what Weight the Leaver will raise of itself.

This cannot be exactly Calculated, unless the Leaver be suppos'd to be of an equal size from end to end:

but if this be granted, then it may be done by this

R U L E.

First find the Weight of the whole Leaver, then that of the shorter end, by saying, as the whole length of the Leaver, is to the whole Weight of it; so is the whole length of the shorter end, to its Weight. Then Divide the Length of the longer end, by that of the shorter end, and reckon the Quotient for the Number of places in a natural Progression. Find the Sum of this Progression, and by it multiply the Weight of the shorter end, and from the Product subtract the Weight of the shorter end, the Remainder is the Weight to be added to the shorter end, to make it Equiponderate with the longer end.

Example.

If the Leaver exemplify'd in Prob. 2 and 3, were (of season'd Oak,) 2 Inches Diameter; its whole Weight will be about 11 pounds; and the Weight of the shorter end about one pound. Therefore (working by the Rule) the Weight to be added to the shorter end, to make it equiponderate with the longer end, will be found to be 54 pounds. So that the bare Weight of this Leaver, thus dispos'd, will arise 54 pounds.

The Method of solving this Problem, I discover'd in August, 1700. I never could see it in any Author; But for the Verity of it, I have (besides Experience) a Demonstration, grounded on the Principles of the Mechanic Powers.

P R O B. 5.

To find the Force of any Axis in Peritrochio, or Wheel.

The R U L E.

As the Semidiameter of the Axis or Spindle, is to the Semidiameter of the Wheel; so is 1, to the Force of that Wheel.

Example.

Example.

If the Diameter of the Axis be 6 Inches, and the Diameter of the Wheel 5 Feet; What will be the Force of that Wheel? Ans. 10 Fold, (for, as 3, is to 30; so is 1 to 10.) or a Power equivalent to one Pound, apply'd to the Circumference of the Wheel, will raise a Weight of 10 Pound, apply'd to the Circumference of the Axis, or one Man with this Mechanic Power, will be as strong as 10 without it.

P R O B. 6.

To find what Power will be sufficient, to raise a given Weight, with a given Wheel.

The R U L E.

As the Semidiameter of the Wheel, is to the Semidiameter of the Axis; so is the Weight to the Power.

Example.

What Power, will be sufficient to raise a Weight, of 1000 Pound, with the Wheel exemplify'd in Prob. 5. Ans. A Power equivalent to 100 Pound.

P R O B. 7.

To find what Weight may be rais'd, by a given Power, with a given Wheel.

The R U L E.

As the Semidiameter of the Axis, is to the Semidiameter of the Wheel; so is the Power, to the Weight.

Example.

What Weight may be rais'd by a Power equivalent to a 100 Pound, with the Wheel exemplify'd in Prob. 5. Ans. 1000 Pound.

P R O B.

P R O B. 8.

To find the Force of any Number of Pulleys.

Before I shew how to solve this Problem, it may be necessary to Premise,

1. That one or more Pulleys, whose Snatch-Blocks are immovable, neither Increase nor Diminish the Power. Hence,

2. All the upper Pulleys, do neither Increase nor Diminish the Power; but serve only for Conveniency. This being Premis'd; this Problem may be solv'd by these

R U L E S.

1. If the end of the Rope be fastned to the upper immovable Snatch-Block; then say, as 1, is to 2; so is the Number of the lower Pulleys, to the Force of those Pulleys. But

2. If the Rope be fasten'd to the movable Snatch-Block; then add one to the 4th Proportional found by the last Rule, so have you the Force of those Pulleys.

Example.

If there be 4 Pulleys, viz. two fixed above and two movable below; What is the force of those Pulleys? Ans. (if the Rope be fixed to the upper Snatch-Block) 4 fold, (for, as 1, is to 2; so is 2 to 4,) or one Man, with this Engine may raise a Weight four times as big as he can do without it.

But, if the Rope be fasten'd to the lower Snatch-Block of these Pulleys; then will their Force be 5 Fold; or one Man, by their help, shall be as strong as five Men without their help.

P R O B.

P R O B. 9.

To find what Power will be sufficient to raise a given Weight with a given Number of Pulleys.

The R U L E.

If the Rope be fastened to the $\left\{ \begin{smallmatrix} \text{upper} \\ \text{lower} \end{smallmatrix} \right\}$ Snatch-Block say, as $\left\{ \begin{smallmatrix} \text{double the Number of lower Pulleys,} \\ \text{double} \text{ ————— more by one,} \end{smallmatrix} \right\}$ is to 1; so is the Weight, to the Power.

Example $\left\{ \begin{smallmatrix} 1. \\ 2. \end{smallmatrix} \right\}$

What Power will be able to raise a Weight of $\left\{ \begin{smallmatrix} 400 \\ 500 \end{smallmatrix} \right\}$ Pound, with two lower Pullies, the Rope being fasten'd to the $\left\{ \begin{smallmatrix} \text{upper} \\ \text{lower} \end{smallmatrix} \right\}$ Snatch-Block? Ans. A Power equivalent to 100 Pound. For ———

As $\left\{ \begin{smallmatrix} 4, \text{ is to } 1; \text{ so is } 400 \text{ to} \\ 4 + 1 = 5, \text{ is to } 1; \text{ so is } 500 \text{ to} \end{smallmatrix} \right\} 100.$

P R O B. 10.

To find what Weight may be rais'd by a given Power, with a given Number of Pulleys.

The R U L E.

First find the Force of the given Pulleys (by Prob. 8.) then Multiply the given Power thereby; the Product is the Weight that may be rais'd by that Power.

Example.

Example $\left\{ \begin{array}{l} 1. \\ 2. \end{array} \right.$

If the Rope be fastened to the $\left\{ \begin{array}{l} \text{upper} \\ \text{lower} \end{array} \right.$ Snatch-Block ;
What Weight may be rais'd by a Power equivalent to
100 Pound, with two lower Pulleys? Ans. $\left\{ \begin{array}{l} 400 \\ 500 \end{array} \right.$
Pound.

Note 1. The Increase of Power by Pulleys hitherto
treated of, proceeds only according to Arithmetical Pro-
gression, beginning at $\left\{ \begin{array}{l} 2 \\ 3 \end{array} \right.$ if the Rope be fasten'd to
the $\left\{ \begin{array}{l} \text{upper} \\ \text{lower} \end{array} \right.$ Snatch-Block : But Pulleys may be
dispos'd in another manner ; so that the Force of the
Power shall be Doubled for one Pulley, Quadrupl'd for
two, Octupl'd for three ; and if there be four Pulleys
the Power will equal to 16, and so of the rest. But the
particular Method of *this*, I shall at present pretermitt.

P R O B. II.

To find the Force of any Wedge.

Some Wedges are made Wedge-wise in thickness, others
in breadth. In the $\left\{ \begin{array}{l} \text{former} \\ \text{latter} \end{array} \right.$ Case, say as the
 $\left\{ \begin{array}{l} \text{Thickness} \\ \text{Breadth} \end{array} \right.$ of the Wedge at the $\left\{ \begin{array}{l} \text{thick} \\ \text{broad} \end{array} \right.$ end, is to
its Length ; so is 1, to the Force of that Wedge.

Note. This Rule supposes the Wedges to Terminate in
a Point, but sometimes Wedges of the later kind, do
not so. In this Case, Subtract the breadth of the narrow
end, from that of the broad end ; then say, as that Re-
mainder, is to the length of the Wedge ; so is 1, to its
Force.

Example.

If a Wedge be two Inches broad, and 12 Inches long,
termi-

terminating in a Point; What is the Force of that Wedge? Ans. 6 Fold, (for, as 2, is to 12; so is 1 to 6.) or one Man shall be as strong with this Wedge, as 6 Men without it.

Note. The Power moving a Wedge, is Percussion; which is a thing that cannot be sufficiently explain'd in few Words, and therefore I shall at present omit it.

P R O B. 12.

To find the Force of any Skrew.

The R U L E.

As the Distance of the Helixes or Threads, is to the Circumference of the Circle describ'd by the Handle, or what else you will call it) whereby the Skrew is turn'd about; so is one, to the Force of that Skrew.

Example.

If the distance of the Helixes be $1\frac{1}{2}$ Inch, and the Handle (whereby the Skrew is turn'd about) stick out $3\frac{1}{2}$ Feet from the Centre of the Skrew; What is the Force of that Skrew? Ans. 176 Fold, (for the Circumference of the Circle describ'd by the Handle, will be 22 Feet or 264 Inches, therefore, as 1,5 is to 264; so is 1, to 176.) or one Man with this Skrew, will be as strong as 176 Men without it. Or, supposing the Strength of one Man, but equal to a 100 Weight or 112 Pound, he may (with this Skrew) raise a weight (or press with a Force) equal to eight Tuns and 16 Hundred Weight; So prodigious is the Force of this Mechanic Powers.

P R O B. 13.

To find the Force of (or rather Advantage gain'd by) any Inclining Plain.

The R U L E.

As the Perpendicular Height of any Inclining Plain,

is to its length; so is one, to the Advantage gain'd by that Inclining Plain.

Example.

If the Height an Inclining Plain be 2 Feet, and its Length 8 Feet; What Advantage (in raising a Weight) is gain'd thereby? Ans. A 4 Fold advantage; Or one Man may raise a Weight to the top of that Inclining Plain, as easily as four Men can raise the same Weight, to the same Height without the Inclining Plain.

P R O B. 14.

To find the Height and Length of an Inclining Plain, by the help of which, a given Weight may be rais'd to a given Height, by a given Power.

The R U L E.

The Height must be the same with the Height the Weight is to be rais'd. And for the length, say, as the given Power, is to the given Weight; so is the Height of the Inclining Plain, to its length.

Example.

If it be requir'd to raise a Weight of 400 Pound, to the Height of 4 Feet, with a power equivalent to 100 Pound; What must be the Height and length of the Inclining Plain, Ans. The Height must be 4 Feet, and the Length 16 Feet.

The Engines of Strength (in Lifting, Pressing and Drawing,) that have been, or may be contriv'd, from the Composition of two or more of these Mechanic Powers, are so many, that I have not room here, so much as to mention them. I shall therefore conclude this Chapter with the following

P R O B. 15.

To find the Force of any Machine or Engine, compos'd of two or more of these Mechanic Powers.

The

The R U L E.

First find the Space or Distance mov'd by the Power, in the same time as the Weight is remov'd any other Space; (which is easily found by comparing the several Parts of the Engin;) then say, as the Motion of the Weight, is to the Motion of the Power; so is 1, to the Force of that Engin.

Example.

In the Engin commonly call'd the Perpetual Skrew, and Axis in Peritrochio; let the Diameter of the Axis be $3\frac{1}{2}$ Inches, and the Diameter of the Wheel $3\frac{1}{2}$ Feet; let this Wheel have 50 Teeth, and the Skrew to make one Revolution for each Tooth; and let the length of the Handle or Winch, (by which the Skrew is turn'd about) be 14 Inches; What is the Force of this Engin? Ans. 400 Fold. Or one Man with this Engin, shall be as strong as 400 Men without it. For while the Weight is rais'd the space of 11 Inches, (equal to the Circumference of the Axis) the Power (at the Winch) will have mov'd 4400 Inches, (for the Winch makes 50 Revolutions, while the Wheel makes one, and each Revolution of the Winch is 88 Inches;) therefore I say, as 11 the Motion of the Weight, is to 4400 the Motion of the Power; so is 1, to 400, the Answer to the Question.

C H A P. VIII.

*Of Automaticks or Clock-Work.*P R O B. $\begin{cases} 1. \\ 2. \end{cases}$

The Diameter of any $\begin{cases} \text{Pinion,} \\ \text{Wheel,} \end{cases}$ being given, with the Number of its $\begin{cases} \text{Leaves} \\ \text{Teeth,} \end{cases}$ and the Number of $\begin{cases} \text{Teeth} \\ \text{Leaves} \end{cases}$ in the $\begin{cases} \text{Wheel} \\ \text{Pinion,} \end{cases}$ that it is to work into, being also given;
To find the Diameter of that $\begin{cases} \text{Wheel.} \\ \text{Pinion.} \end{cases}$ (A 2) The

The Rule. As the $\left\{ \begin{array}{c} \text{Leaves} \\ \text{Teeth,} \end{array} \right\}$ of the $\left\{ \begin{array}{c} \text{Pinion} \\ \text{Wheel,} \end{array} \right\}$ is to the $\left\{ \begin{array}{c} \text{Teeth} \\ \text{Leaves} \end{array} \right\}$ of the $\left\{ \begin{array}{c} \text{Wheel;} \\ \text{or Pinion;} \end{array} \right\}$ so is the Diameter of the $\left\{ \begin{array}{c} \text{Pin.} \\ \text{Wh.} \end{array} \right\}$ to the Diameter of the $\left\{ \begin{array}{c} \text{Wheel.} \\ \text{Pinion.} \end{array} \right\}$

Example.

If the Diameter of a Pinnion, or Wheel be .5 or $\frac{1}{2}$ an Inch, or 4 Inches, and have 5 Leaves, or 40 Teeth; and if it be requir'd to fit to it a Wheel of 40 Teeth, or Pinion of five Leaves; what must be the Diameter of that Wheel or Pinion? Ans. 4 Inches, or .5 of an Inch.

P R O B. 13.

To find the Number of Beats made in an Hour, (or Minute Secs.) by any Watch.

The R U L E.

Divide (the Number of Teeth in) every Wheel, (from the great Wheel or Fusy, to the Crown Wheel, or Swing Wheel,) by the (number of Leaves in the) Pinion it drives; and multiply all the Quotients one into another continually, and the last Product by the Number of Teeth in the Crown (or Swing) Wheel; and multiply this last Product by two (for the Pallats;) this last Product gives you the Number of Beates in one turn of the great Wheel or Fusy.

Then divide the Dial-Wheel by the Pinion of Report, and by the Quotient multiply the Number of Beats (but now found) in one Turn of the Hand or 12 Hours; Therefore, dividing that Product by 12, the Quotient is the Beat in an Hour: And if this Quotient be divided by 60, the Quotient will be the Beats in a Minute: And if this last Quotient be again Divided by 60 the Quotient will be the Beats in a Second.

Example.

Example.

Let there be a Watch of 4 Wheels, whose Teeth are according to the following Numbers.

First, The Great Wheel, (Number 55 Teeth) turning the Pinion (Number 5.) fix'd to the Arbor of the second Wheel.

Secondly, The second Wheel, (Number 45,) turning the Pinion (Number 5,) fix'd to the Arbor of the Contrat Wheel.

Thirdly, The Contrat-Wheel, Number 40,) turning the Pinion (Number 5,) fix'd to the Crown Wheel.

Fourthly, The Crown Wheel, Number 17, working upon the Pallats, Number 2.

Thus for Watches of 4 Wheels; but for Watches of five Wheels, there will be a third Wheel before the Contrat Wheel.

Fifthly, The Pinion of Report fix'd to the Arbor of the Great Wheel, (Number 4,) turning the Dial-Wheel (Number 36,) which carries the Hand about on the Face in 12 Hours.

Now, let it be requir'd to find how many Beats this Watch makes in an Hour, or a Minute. To answer this Question, I work by the Rule, dividing each Wheel by its Pinion, and find the Quotients to the three first Wheels (divided by their Pinions) to be 11, 9, 8. these multiply'd together produce 792, (which is the Number of Turns of the Crown-Wheel, to one Turn of the Great Wheel;) which being multiplied by 17, (the Number of Teeth in the Crown Wheel, and that Product by two, (the Pallats) produces 26928, (the Beats in one turn of the Great Wheel. (Then, the Dial-Wheel divided by the Pinion of Report, the Quotient is 9, by which I multiply 26928, (the Beats in one turn of the Great Wheel, and the Product is 242352, the Beats in (one turn of the Hand, or) 12 Hours, which being divided by 12, gives 20196, the Beats in an Hour, which being divided by 60, gives 336.6, that

is somewhat above $336\frac{1}{2}$, the Beats in a Minute.

P R O B. 4.

*The Beats and Time of a Watch's going, being propos'd;
To find what Number of Turns the Fusy will have.*

The R U L E.

As the Beats in one Turn of the Great Wheel or Fusy, (suppose, as in the last Example, 26928,) is to the Beats in one Hour; suppose as there also, 20196,) so is the continuance of the Watch's going in Hours, (suppose 16,) to the Number of the Turns of the Fusy, (12.)

P R O B. 5.

By the Beats, and Turns of the Fusy; To find how many Hours the Watch will go.

The R U L E.

As the Beats in one Hour, (suppose 20196) is to the Beats in one Turn of the Fusy; (suppose 26928) so is the Number of the Turns of the Fusy, (suppose 12,) to (16,) the continuance of the Watch's going in Hours.

P R O B. 6.

By the Turns of the Fusy, and continuance of the Watch's going in an Hour; To find the Beats in one Turn of the Fusy.

The R U L E.

As the Number of Turns of the Fusy. (suppose 12,) is to the continuance of the Watch's going in Hours; (suppose 16,) so are the Beats in an Hour, (suppose 20196,) to (26928,) the Beats in one Turn of the Fusy.

P R O B. 7.

*By the Time of the Watch's going, the Number of Turns of the Fusy, and the Beats in one Turn of the Fusy;
To find the Beats in an Hour.*

Th

The R U L E.

As the continuance of the Watch's going in Hours, (suppose 16,) is to the Number of Turns of the Fusy (suppose 12,) so are the Beats in one Turn of the Fusy (suppose 26928,) to 20196,) the Beats in an Hour.

P R O B. 8.

By the Beats in one Turn of the Fusy, and the Beats in an Hour, and the Hours of the Face, (which are always 12, or 24.) To find what Quotient must be laid upon the Pinion of Report.

The R U L E.

As the Beats in one turn of the Great Wheel or Fusy; (suppose 26928,) is to the beats in an Hour (suppose 20196,) so is the Hours of the Face, (suppose 12,) to (9,) the Quotient of the Hour-Wheel (or Dial-Wheel) divided by the Pinion of Report, i. e. the Number of Turns of the Pinion of Report, in one Turn of the Dial-Wheel.

P R O B. 9.

Another way to find what Quotient must be laid upon the Pinion of Report; by knowing the time of the Watch's going, the Turns of the Fusy, and the Hours of the Face.

The R U L E.

As the Hours of the Watch's going, (suppose 16,) are to the Turns of the Fusy (suppose 12,) so are the Hours of the Face, (suppose 12,) to (9,) the Quotient of the Pinion of Report.

P R O B. 10.

To find the exact length of a Pendulum, for any Clock, or Watch.

The R U L E.

First find (by Prob. 3.) the Number of beats in a Minute. Then say, by Duplicate Proportion, (v. Book 1. Chap. 15.) As the beats in a Minute, is to 60; so is 39.2; to the length of the Pendulum in Inches.

Otherwise,

Say, (by the Golden Rule) as the Square of the beats in a Minute, is to (the Square of 60,) 3600; so is 39.2, to the length of the Pendulum in Inches.

Example.

If a Watch make 120 beats in a Minute; how long must its Pendulum be? Ans. 9.8 Inches, (i. e. somewhat above 9 $\frac{1}{2}$ Inches.) For, as 14400 is to 3600; so is 39.2 to 9.8.

P R O B. 11.

To find the Beats in a Minute; by knowing the length of the Pendulum.

The R U L E.

Say, by Subduplicate Proportion, (v. Book I. Chap. 15.) As the length (in Inches) given, is to 39.2; so is 60, to the Beats in a Minute.

Otherwise,

Say, (by the Golden Rule,) as the length (in Inches) given, is to 39.2; so is (the Square of 60,) 3600, to the Square of the Beats in a Minute.

Therefore, if the length of the Pendulum be propos'd, the Numbers (for the Wheels of the Watch) must be so adapted, as to make the Number of Beats in a Minute found by this Rule.

P R O B. 12.

To Calculate the Striking Part of a Clock.

The R U L E.

Though this part consists of many Wheels, and Pinnions; yet respect need be had only to the Count-Wheel, Striking-Wheel, and Detent-Wheel, which move round in this Proportion. The Count-Wheel commonly moves round once in 12, (sometimes in 24,) Hours. The Detent-Wheel moves round commonly every stroke the Clock Strikes) sometime but once in two Strokes.) Hence it follows,

1. That

1. That, as many Pins as are in the Pin-Wheel, so many Turns hath the Detent-Wheel to one Turn of the Pin-Wheel. Or, (which is the same thing) the Number of Pins in the Pin-Wheel, are equal to the Quotient of that Wheel divided by the Pinion of the Detent-Wheel. But if the Detent-Wheel moves but once round in two Strokes of the Clock, then the said Quotient is but half the Number of Pins.

2. As many Turns of the Pin-Wheel as are requir'd to make the Strokes (which are 78) of 12 Hours, so many Turns must the Pinion of Report have to turn round the Count-Wheel once. Or thus, divide 78 by the Number of Striking Pins, and the Quotient shall be the Quotient of the Pinion of Report. All this is, in case the Pinion of Report (as is very common) be fix'd to the Arbor of the Pin-Wheel.

These following Rules will be of great use in this part of Calculation, viz.

R U L E. 1.

As the Number of Turns of the Great Wheel or Fusy, is to the Days of the Clock's Continuance or going; so is (156) the Number of Strokes in 24 Hours, to the Strokes in one Turn of the Great Wheel or Fusy.

R U L E. 2.

As (156) the Number of Strokes in 24 Hours, is to the Strokes in one Turn of the Fusy; so are the Turns of the Fusy, to the Days of the Clock's Continuance.

R U L E. 3.

As the Strokes in one turn of the Fusy, is to (156,) the Strokes in 24 Hours; so is the Days of the Clock's Continuance, to the Number of Turns of the Fusy.

R U L E. 4.

As the Number of Strokes in the Clock's Continuance, or in all its Turns of the Fusy, is to the Turns of the Fusy; so is (78,) the Strokes in 12 Hours, to the Quotient of the Pinion of Report, fix'd to the Arbor of the Great Wheel.

P R O B.

PROB. 12.

By knowing what Weight will move the wheels of a Clock, for one time; To find what Weight will be requir'd to make the same Clock go any other time, keeping the same Descent.

The RULE.

As the time of the Clock's going, given, is to the time requir'd; so is the weight given, to the weight sought.

Example.

Suppose a common Clock, that requires a weight of 50 Pounds, to keep it in Motion; and suppose the Place where the weight descends be 20 Feet, and that this Descent serves for 12 Hours. Let it be requir'd (with the same Descent of the weight) to make this Clock go 24 Hours, (which may be done by adding a single Pulley to the weight;) what weight (in this Case) must the Clock have? Answ. 100 Pound. For, as 12, is to 24; so is 50, to 100.

CHAP. IX.

Of Philosophy.

PROB. 1.

To find the Strength of any Square piece of Timber.

The RULE.

As the length of it in Feet, is to the Square of its thickness in Inches, multiply'd by its breadth in Inches, so is the Strength (viz. how much weight it will bear, when dispos'd Horizontally, and supported by the ends with convenient Blocks or Fulcras, and the weight hang'd in the middle) of a piece of Timber (of the same kind) one Inch Square, and one Foot long, to the Strength of given piece.

Note.

Note 1. That by Thickness I here mean that way of the Timber which is to bear the Strefs. Thus, if a piece of Timber (which is broader than 'tis thick) be laid Edge-wise, then the breadth of it is accounted the thickness; and so it will bear a much greater weight, than if the same piece were laid the flat way.

2. That this Rule is general for all kind of Timber; if the Strength (*viz.* What weight it will bear) of a piece an Inch Square and a Foot long (of each kind of Timber) be first known, or found by Experience.

3. That the Strength of a piece an Inch Square and a Foot long, being known; it must be the third Term in the foregoing Analogy.

Example 1

Suppose the Strength of a piece of Oak, an Inch Square and a Foot long, to be (but) 200 weight, (tho' I am sure it will be a great deal more;) what will a Joynt bear, that (lying Edge-wise) is 8 Inches thick, and 3 Inches broad, and 10 Feet long? *Ans.* 38.4 Hundred weight, or one Tun, and 18.4 Hundred weight.

P R O B. 2.

*Any heavy Body being propos'd to be let fall from on high;
To find how far it will descend in any given Time.*

The R U L E.

Reduce the given Time into Seconds of an Hour; Then say, as 1, is to the Square of those Seconds; so is 12.5, to the Space or Distance (in Feet English) it will descend in that time.

Example.

How far will a falling Body descend in one Minute of an Hour? *Ans.* 45000 Feet, that is, 120 Feet above 8 $\frac{1}{2}$ English Statute Miles.

P R O B.

P R O B. 3.

Any Height being propos'd, from whence a Body is to be let fall; To find how long it will be a falling.

The R U L E.

As 12.5, is to the given Height in Feet; so is 1, to the Square of the Seconds of the Time that the Body will be a falling.

Example.

How long will a Body be a falling, from the Height of one English Mile? Answ. 20.5 Seconds, that is a very little above $\frac{1}{3}$ of a Minute.

P R O B. 4.

Any Time being propos'd; To find the Space or Distance that a Sound will fly in that Time.

The R U L E.

Reduce the given time into the Seconds of an Hour. Then say, As 1, is to 1400, or to 84.8484; so is the given Time in Seconds, to the Distance in Feet, or Perches, that a Sound will fly in that Time.

Otherwise,

Without reducing the given Time into Seconds, work thus: If the Time be given, in Minutes or Hours, say, as 1, is to 15.9, or to 954.54; so is the given time in Minutes, or Hours, to the Distance in English Miles, that a Sound will fly in that time.

Example 1.

How far will Sound fly in a Minute of an Hour? Answ. 15.9 Miles, that is almost 16 Miles.

Example 2.

How far will Sound pass in an Hour? Answ. 954.54 Miles i. e. somewhat above 954 Miles and a half.

P R O B.

P R O B. 5.

Any Space or Distance being propos'd; To find how long Sound will be flying that Distance.

The R U L E.

If the Space or Distance propos'd, exceed not 15 English Miles; you may say, as 1400, or 84.8484, is to 1, so is the Distance propos'd in Feet, or Perches, to the Time in Seconds, that a Sound will require to fly that Distance.

But, if the Distance propos'd exceed 15 Miles; you may say, as 15.9, or 954.54, is to 1; so is the Distance propos'd in Miles English, to the Time in Minutes, or Hours, that a Sound will require to fly that Distance.

Example.

How long will Sound be a flying 40 Miles? Ans. 2.51 Minutes, *i. e.* a little above two Minutes and a half.

Note 1. That this Chapter is wholly new, none of it (to my knowledge, being ever Publish'd before.

2. That the Grounds of these 5 Philosophical Problems, (and many more, which to my knowledge were never made Public,) together with a larger Account of them, and many Curiosities relating to them, the Reader may find in my *Philosophia Practica*; or *Philosophical Experiments and Observations reduced to Practice*; which I may shortly present the World with, if this find but a kind Reception. And therefore I am unwilling to anticipate that Treatise, by transcribing any more Propositions from it. But I shall at this time let the Reader find

F I N I S.

Postscript.

Courteous Reader,

THOUGH in this Treatise, I promis'd only, to shew how to solve all the *Propositions* therein contain'd, by *Gunter's-Line*; yet (I have been better than my Word; for) as I have *here* propos'd them Analogically, they may as well be solved by the *Pen*; or by any other *Instrument* whereby an Analogy may be wrought; of which kind of Instruments there are very many: And therefore this Book (being an Epitome of the whole *Practical Mathematicks*) may be a fit Pocket-Companion for any Man, of what Degree or Profession soever.



V A L E

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